Lancaster Particle Pamphlets

Number 2 Accelerators

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Particle Accelerators

Introduction

The advances in particle physics made over recent years have resulted from the construction of large particle accelerators. An accelerator can be regarded as a "microscope" for the study of the fundamental constituents of matter. The *smaller* the length involved the *larger* the accelerator has to be. Large accelerators are also required to create some of the fundamental building blocks of matter, for example the top quark has a mass two hundred times the mass of a single proton.

The resolving power of an optical microscope is limited by the wavelength of the light used. Visible light has a wavelength of about 500 nm and objects smaller than this cannot be seen in detail. To study what goes on inside a single proton, we need a "microscope" that can resolve distances much less than 10^{-15} m.. Remember that particles have a wave characteristic, and the De Broglie relation gives the wavelength $l = h/p$, where p is the momentum and h is the Planck constant. For example, an electron with energy 1 GeV has $l \approx 10^{-15}$ m. Notice that the wavelength is inversely proportional to the momentum. Smaller wavelength requires higher momentum and consequently higher energy. We require electrons with energy much greater than 1 GeV to study the internal structure of the proton. The Stanford "two mile" linear accelerator at Stanford USA accelerates electrons to 20 GeV; it was used to establish the quark substructure of protons and neutrons.

The largest accelerator now being constructed at the CERN laboratory in Geneva will accelerate protons to 7000 GeV. This will indeed be a very large microscope; it is contained in a tunnel approximately 21 km in circumference.

Theory

The working of an accelerator depends crucially on two bits of physics that should be familiar to you. These are:

- The motion of a charged particle in an electric field *E*.
- The motion of a charged particle in a magnetic field *B*.

The function of the electric field is to accelerate the charged particle and increase its energy. The function of the magnetic field is to bend the particle trajectory and keep it in a circular orbit.

Motion in an electric field

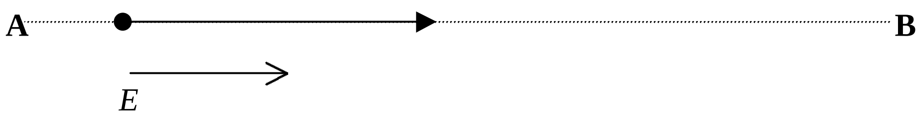


Figure 1

Imagine a particle, charge q (Coulombs) moving from A towards B. A uniform electric field E is applied in a direction parallel to the particle's velocity as shown in Fig 1. The electric field exerts a force $F = qE$ on the particle. The direction of the force is parallel to the velocity so the work done on the particle as it moves from A to B through distance d is:

$$W = Fd = qEd$$

We can express this formula in terms of the *decrease* in electrical potential over the distance d. This decrease is given by $V = Ed$ so that

$W = qV$

The work done increases the particle's kinetic energy. If the total energy of the particle is E_A at A and is E_B at B then conservation of energy gives:

$$E_B = E_A + qV \qquad (\text{J}) \qquad (1)$$

The standard unit for energy is the Joule but in particle physics it is more convenient to use the units eV or GeV. (1 GeV=1.6×10^{-10} J) When a particle charge q passes through an electrical potential drop of V volts then the *gain* in energy is qV Joules or qV/e eV, where e is the magnitude of the electron charge in Coulombs. If we express the particle charge in terms of the electron charge e, $Q = q/e$, then equation (1) takes the form:

$$E_B = E_A + QV \quad (\text{eV}) \quad \text{with } Q = q/e \qquad (2)$$

An example illustrates the point. A single proton has positive charge equal in magnitude to the electron charge, so it has $Q = +1$. If a proton is accelerated through a potential drop of 5.0×10^9 V then its total energy is increased by 5.0×10^9 eV or 5 GeV. As we will see, such large potential drops are readily achieved in modern accelerators.

[Remember that with increases in energy much greater than the rest mass energy the relativistic formulae discussed in P1 must be used.]

Motion in a magnetic field

Imagine another particle with charge *q* initially moving with velocity *v* at point A. A uniform magnetic field *B* is applied so that the direction of *B* is perpendicular to *v* and *into* the page. This is shown in Fig 2.

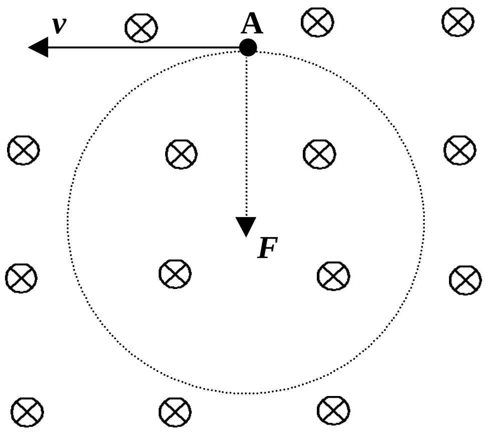


Figure 2

When the magnetic field *B* and the velocity *v* are perpendicular to each other then the resulting force *F* on the charged particle is perpendicular to both. The direction of *F* when the particle is at A is shown in Fig 2. The magnitude of *F* is given by the formula:

$$F = qvB$$

The resulting motion of the particle is a circle radius *r* (shown dotted in Fig 2). The magnetic force gives the *centripetal acceleration* that keeps the particle in its circular orbit. Using Newton's second law (force = mass × acceleration):

$$qvB = mv^2 / r$$

so that $qB = mv / r$

It is convenient to rewrite this formula in terms of particle momentum $p = mv$:

$$qB = p/r \text{ or } p = qBr$$

In this form, the equation is still true at high energies when relativity effects are important. We can make further useful simplifications by changing to GeV/c units for momentum and expressing the charge in units of the magnitude of the electron charge. The result is:

$$p = 0.3QBr \text{ GeV/c} \quad (3)$$
with B in Tesla and r in metres.

As an example consider a proton ($Q = 1$) with momentum p = 40 GeV/c moving perpendicular to a magnetic field of 1.5 T. What is the radius of curvature of its trajectory? From equation (3) we get:

$$r = \frac{p}{0.3B}$$
$$= \frac{40}{0.3 \times 1.5} = 89 \text{ m}$$

A field of 1.5 T is very strong and the result gives some idea of the size of modern accelerators. If the momentum were 400 GeV/c the radius would be 889m and at 4000 GeV/c it would be close to 10 km.

Our equations also allow us to calculate the time taken for a particle to complete a full turn through 360° in a uniform magnetic field. Starting with the expression $qB = mv/r$ we find:

$v = qBr/m$
let t be the time for a complete revolution
$t = 2pr/v = 2prm/qBr = 2pm/qB$
the frequency of revolution f is then
$f = 1/t = qB/2pm$

This frequency is called the "*cyclotron frequency*" and you will see that it is independent of the orbit radius. If we put $q = 1.6 \times 10^{-19}$ C, $B = 1$T and $m = 1.67 \times 10^{-27}$ kg then $f \approx 15$ MHz .

This frequency is in the Radio Frequency range and sets the stage for accelerator calculations. However if the particle's speed is significant fraction of the speed of light then we use the expression $m = g\, m_0$ for the mass. The cyclotron frequency then decreases as the gamma factor increases. (refer to P1 for a discussion of the relativistic gamma factor)

Accelerator layout

The early cyclotron

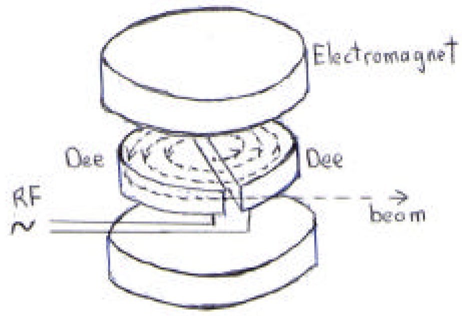


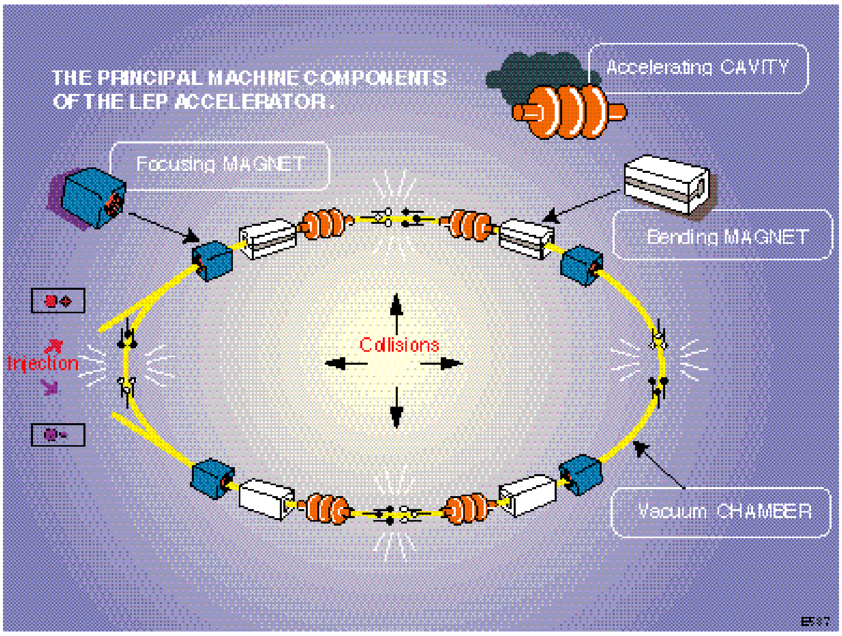
Figure 3

Figure 3 shows a schematic diagram of the earliest circular particle accelerator, the cyclotron. Lawrence invented it in 1932 and received a Nobel prize. Ionised hydrogen nuclei (protons) are introduced at the centre of the two D-shaped cans called “dees”. The dees are placed between the poles of a powerful electromagnet that produces a uniform field of about 1 T. An alternating potential is applied to the dees at the appropriate cyclotron frequency. The protons are accelerated towards the negatively charged dee, bend through 180° and are accelerated again to the other dee since the polarity has switched in phase with the rotation frequency. Bunches of protons are accelerated simultaneously and spiral outwards until they reach the maximum radius. An additional electrode or magnet (not shown) then kicks the proton bunch out of the machine and into the experimental area.

The energy achieved by cyclotrons is limited by the cost of building huge electromagnets. A practical limit is of the order of several hundred MeV. Improvements in technology allow a much more practical solution to the problem of achieving very high energies.

The Modern Synchrotron

In the modern **synchrotro**n the accelerated particles are held in a circular orbit of *constant radius*. This has the immediate advantage that the magnetic field need only be produced in the region of this orbit. The cost of a cyclotron magnet increases as the *square* of the maximum orbit radius whereas the cost of the synchrotron magnet is proportional to its radius. The magnets that keep the particles in orbit are called “dipoles” and the cross section of one of them is shown in figure 4. The particles are accelerated by alternating electric fields in radio frequency (RF) cavities placed at several locations around the ring. As the particles gain energy their momentum increases and the strength of the magnetic field is increased in proportion (equation 3). An important feature of the synchrotron is that provided the magnetic field is increased slowly, bunches of particles automatically stay in phase with the frequency of the accelerating electric field. This "phase stability" is the origin of the synchrotron name.



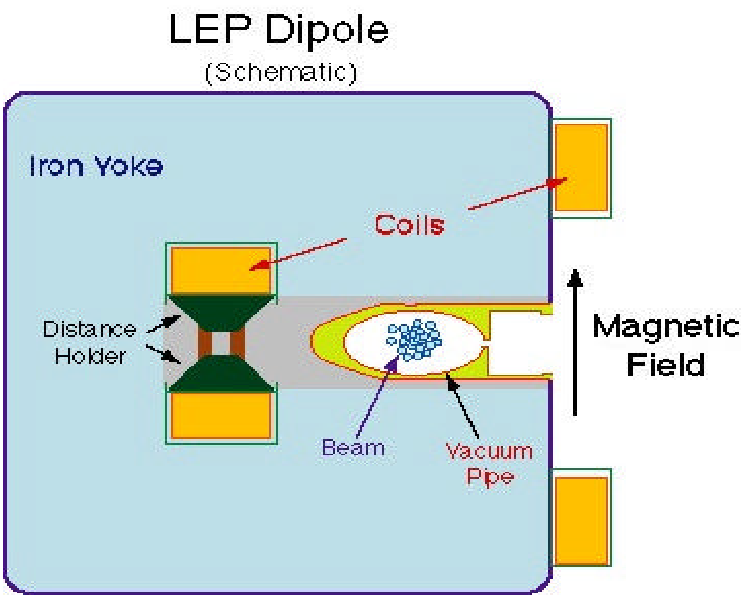


Figure 4

A potential problem with synchrotrons is the tendency of charged particles to oscillate with large amplitudes around the optimum orbit path (so-called betatron oscillations). If these oscillations are large then the magnets have to have large gaps between the pole tips and hence cost too much. Quadrupole magnets placed at regular intervals around the ring control the amplitude of the oscillations. These behave rather like the positive and negative lenses in the telephoto lens of a camera. Strong converging magnets are alternated with strong diverging magnets around the ring. The combined effect is to minimise the amplitude of the beam oscillations whilst still maintaining a net focussing action.

The beam itself is contained in a narrow pipe maintained under very high vacuum conditions. In a typical modern synchrotron there are thousands of dipole magnets around the ring with associated smaller quadrupoles. A typical “cell” containing two dipole bending magnets and two quadrupoles, one strongly focusing and the other strongly defocusing, is shown schematically in figure 5.

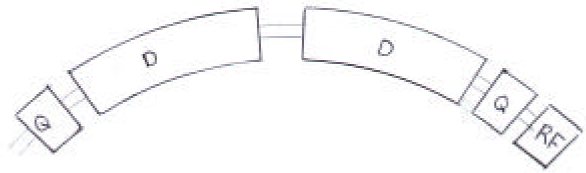


Figure 5

The basic principles of acceleration and guidance are the same in a modern synchrotron as in the early cyclotron. However the invention of strong focusing and defocusing elements and also major improvements in control equipment and precision engineering have allowed enormous increases in beam energy. The early Lawrence cyclotron produced particles of about 1 MeV kinetic energy, the LHC (Large Hadron Collider) will produce particles of 7000 GeV.