Macroscopic fluctuations in the boundary resistance of metal-superconductor interfaces

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We examine the possibility of macroscopic fluctuations in the electrical resistance R of a disordered mesoscopic superconductor. A perturbative argument shows that in the presence of phase fluctuations, quantum interference can cause the Andreev scattering amplitude to vanish, leading to a $1/R^2$ tail in the probability distribution for R. Consequently no moments exist and the boundary resistance exhibits large fluctuations. This prediction is confirmed by numerical simulations on superconducting loops, connected to external reservoirs by point contacts.

While transport in low-dimensional, normal materials is now relatively well understood, electronic properties of mesoscopic superconductors and hybrid normal-metal–superconductor (NS) structures remains largely unexplored. Recent predictions of new phenomena, such as conductance fluctuations in superconducting point contacts[1,2] and superconductivity-induced Anderson localization[3,4] suggest that this area of research is likely to yield many new surprises. In this paper, we investigate the statistical properties of the electrical resistance R of a disordered, short coherence length superconductor. This resistance, which arises when excitations incident on the superconductor from normal metallic contacts are only partially Andreev reflected,[5–7] is of particular interest in layered systems,[8–10] where the absence of a proximity effect allows transport properties to be dominated by quasiparticles. For a long superconductor of length L, it reduces to a sum of two boundary resistances, which approach limiting values as $L\to\infty$. Therefore, one might expect that the resistance distribution obtained from an ensemble of disordered superconductors will be statistically well behaved. In what follows, we examine the resistance of a one-dimensional superconducting chain and a system composed of many chains connected in parallel and demonstrate that, on the contrary, in the presence of phase fluctuations, none of the moments of the resistance distribution exist. This behavior arises because, as demonstrated below, the conductance of a single chain or many statistically independent chains connected in parallel, have a finite probability density of vanishing. Classically, even if the conductance of a single chain can vanish, the probability of obtaining a vanishing conductance when two or more chains are connected in parallel, is zero. Thus, the results reported below are quantum mechanical in origin and reflect the coherent nature of quasiparticle transport in mesoscopic superconductors.

Ideally, to demonstrate the existence of large scale fluctuations, one should present a self-consistent description, in which both the quasiparticle scattering matrix and the order parameter $\Delta(\mathbf{r})$ are determined simultaneously in the presence of disorder and flow. Such a description is beyond the capabilities of current theoretical techniques and, therefore, we adopt a less ambitious approach, in which $\Delta(\mathbf{r})$ is assumed to be given. A great deal is already known about the spatial variation of $\Delta(\mathbf{r})$ in short coherence length superconductors.[11–14] Even in the absence of disorder, the order parameter of such systems can vary on the scale of a coherence length ξ, which may be of order the size of a unit cell. In the presence of inhomogeneities such as grain boundaries, local nonstoichiometries, or disordered tangles of vortex lines, both the magnitude and phase of $\Delta(\mathbf{r})$ are expected to exhibit random spatial fluctuations. Given that such fluctuations exist, it seems reasonable to seek generic features which result from them. In what follows, we demonstrate that the existence of singular moments in the distribution of R is one such feature.

To demonstrate this phenomenon, we examine the statistical properties of R for the case of a zero-temperature disordered superconductor of length L, formed, as shown in Fig. 1, by joining together $M\geq 1$, parallel one-dimensional chains and connecting the nodes at $x=0$ and $x=L$ to one-dimensional, normal, external leads. The mth superconducting chain is described by the Bogoliubov-de Gennes equation,

$$\begin{bmatrix} -\partial_x^2-1+U^{(m)}(x) & \Delta^{(m)}(x) \\ \Delta^{(m)}(x)^* & \partial_x^2+1-U^{(m)}(x) \end{bmatrix} \begin{bmatrix} \psi_p^{(m)}(x) \\ \psi_n^{(m)}(x) \end{bmatrix} = E \begin{bmatrix} \psi_p^{(m)}(x) \\ \psi_h^{(m)}(x) \end{bmatrix}, \tag{1}$$

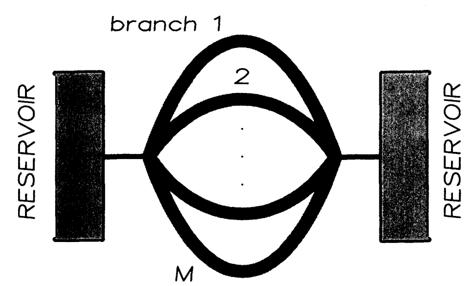

FIG. 1. System composed of M disordered, $1d$ superconducting branches, connected in parallel to perfect, one-channel external leads.

where, for convenience, we choose the Fermi energy E_F as a unit of energy, the inverse Fermi wave vector k_F^{-1} as a unit of length, and measure all energies relative to E_F. The dimensionless order parameter $\Delta^{(m)}(x)$ and normal potential $U^{(m)}(x)$ are random quantities, whose statistical properties are assumed known. The regions $x<0$ and $x>L$ are occupied by one-dimensional external leads, within which $U(x)=\Delta(x)=0$, and the group velocities of particles and holes are $2k$ and $2q$, respectively, where $k^2-1=1-q^2=E$.

For a quasiparticle (quasihole) incident from the left (right), if R_{pp}, R_{hp} (R'_{hh}, R'_{ph}) are reflection coefficients for normal and Andreev scattering, respectively, and $T_{pp}, T_{hp}(T'_{hp}, T'_{ph})$ are corresponding transmission coefficients, then, in units of $h/2e^2$, R can be written[15,16]

$$R=[T_{pp}+T_{hp}+2(R_{hp}R'_{ph}-T_{hp}T'_{ph})/(R_{hp}+R'_{ph}+T_{hp}+T'_{ph})]^{-1} . \tag{2}$$

In the presence of nonvanishing transmission coefficients, R is nonlocal and depends on the form of $\Delta^{(m)}(x)$ and $U^{(m)}(x)$ for all x. However, in the limit $L\to\infty$, where all transmission coefficients vanish, the right-hand side of Eq. (2) can be written as the sum of two interface resistances $R=R_B+R'_B$ of the form,

$$R_B=1/2R_{hp}, \quad R'_B=1/2R'_{ph} . \tag{3}$$

This limiting form of the more general expression (2) was first derived by Blonder, Tinkham, and Klapwijk.[17]

To investigate the possibility of large scale fluctuations in R, we first derive a probability distribution for the Andreev reflection coefficient R_{hp} and use Eq. (3) to derive the distribution of R_B. Since the tail in the distribution of R_B arises from small values of R_{hp}, we start from a normal disordered system and include the effect of $\Delta(x)$ perturbatively. To this end, the matrix on the left-hand side of Eq. (1) is written in the form $H_0^{(m)}+H_1^{(m)}$, where $H_0^{(m)}$ is diagonal and $H_1^{(m)}$ off diagonal. Then, if

$$\begin{bmatrix}\psi_{0p}^{(m)}(x)\\ 0\end{bmatrix}, \begin{bmatrix}0\\ \psi_{0h}^{(m)}(x)\end{bmatrix}$$

are eigenstates of $H_0=\sum_{m=1}^{M}H_0^{(m)}$, corresponding to an incident particle (hole) flux of magnitude $2k$ $(2q)$ from the left in the superconductivity, the first-order contribution to the Andreev scattered, outgoing hole wave function is of amplitude $\psi_{1h}=\sum_{m=1}^{M}\psi_{1h}^{(m)}$, where[16] $\psi_{1h}^{(m)}=\frac{1}{2}\int_0^L|\psi_{0h}^{(m)}(x')|^2\Delta(m)(x')^*dx'$. In this expression, the limit $E=0$ has been taken and we have noted that the unperturbed system possesses both particle-hole and time-reversal symmetry. This yields the Andreev reflection coefficient though the relation $R_{hp}=|\psi_{1h}|^2$. A similar argument for a quasiparticle incident from the right yields R'_{hp}.

If the phase of $\Delta(x)$ fluctuates, the integral contributing to $\psi_{1h}^{(m)}$ has a random sign and ψ_{1h} may vanish with a finite probability density. Hence, when combined with the limiting expression $R_B=1/(2|\psi_{1h}|^2)$, the above result demonstrates that as $L\to\infty$, the possibility of large resistances arises. In the presence of phase fluctuations, to obtain the tail in the distribution of R_B, consider first a given sample with fixed normal potential $U(x)$ and the case where the real and imaginary parts of $\Delta^{(m)}(x)$ are independent Gaussian random processes of mean zero. Since ψ_{1h} is a linear combination of such numbers, its real and imaginary parts are also normally distributed and, consequently, R_{hp} has an exponential distribution of the form $P(R_{hp})=\sigma^{-2}\exp(-R_{hp}/\sigma^2)$. If angular brackets denote an ensemble average over all $\Delta(x)$, then σ^2 is given by $\sigma^2=\langle|\psi_{1h}|^2\rangle$ and depends on the particular realization of the normal potential. For the simplest possible choice of the form $\langle\Delta^{(m)}(x)\rangle=0$ and $\langle\Delta^{(m)}(x)\Delta^{(m')}(x')\rangle=\Delta^2\delta(x-x')\delta_{mm'}$, this yields

$$\sigma^2=(\Delta^2/4)\sum_{m=1}^{M}\int_0^L|\psi_{0h}^{(m)}(x')|^4dx=M\Delta^2/(16\alpha) , \tag{4}$$

where α^{-1} is the average decay length of the localized states $\psi_{0h}^{(m)}(x)$ and the limit $L\to\infty$ has been taken. From $P(R_{hp})$, a straightforward change of variables yields a distribution of R_B of the form $P(R_B)=(2\sigma^2R_B^2)^{-1}\exp(-1/2\sigma^2R_B)$. For large R_B, this approaches a Cauchy distribution, for which all moments diverge. In practice, the form of the distribution at small R_B will be modified, because this region corresponds to large values of R_{hp}, where first-order perturbation theory is no longer valid. Nevertheless, for large R_B, the limiting form $P(R_B)\sim1/(\sigma^2R_B^2)$ is expected to be universal, provided $1/\sigma^2$ is identified with the probability density that R_{hp} vanishes.

The above Gaussian model is, of course, rather artificial. For a real superconductor, $|\Delta(x)|$ and the superfluid velocity $\nabla\theta(x)$ change only on the scale of the coherence length ξ, which, in units of k_F^{-1}, is of the form $\xi=\langle|\Delta(x)|\rangle^{-1}$. To simulate such a variation we have obtained exact numerical results for a system composed of M branches, each divided into N cells of length k_F^{-1}, with the normal potential $U^{(m)}(x)=U_n^{(m)}$ and the order parameter $\Delta^{(m)}(x)=\Delta_n^{(m)}=|\Delta_n^{(m)}|\exp i\theta_n$ in the nth cell of branch m, chosen to be the following random walk:

$$|\Delta_n^{(m)}|=|\Delta_{n-1}^{(m)}|+x_n^{(m)}\langle|\Delta|\rangle^{3/2} , \tag{5}$$

$$\phi_n^{(m)}=\phi_{n-1}^{(m)}+y_n^{(m)}\langle|\Delta|\rangle^{3/2} , \tag{6}$$

$$\theta_n^{(m)}=\theta_{n-1}^{(m)}+\phi_n^{(m)} , \tag{7}$$

$$U_n^{(m)}=z_n^{(m)}\delta U . \tag{8}$$

In these expressions, $x_n^{(m)}$, $y_n^{(m)}$, and $z_n^{(m)}$ are independent random numbers chosen from a uniform distribution with limits ± 1. Checks are performed to ensure that $0<|\Delta_n|<2\langle|\Delta|\rangle$ and that $0<\phi_n<\langle|\Delta|\rangle$. If either of these conditions is not satisfied, further random numbers are substituted for x or y until they are met. After ξ steps, the random walk (5) yields $|\Delta_n^{(m)}|-|\Delta_{n-\xi}^{(m)}| \sim \xi^{1/2}\langle|\Delta|\rangle^{3/2} \sim \langle|\Delta|\rangle$, while Eqs. (7) and (6) combine to yield, $(\nabla\theta_n^{(m)})-(\nabla\theta_{n-\xi}^{(m)}) \sim \phi_n^{(m)}-\phi_{n-\xi}^{(m)} \sim \langle|\Delta|\rangle \sim v_c$, where v_c is the Landau velocity divided by the Fermi velocity. The restriction $0<\phi_n$ ensures that the current cannot change sign, while the condition $\phi_n<\langle|\Delta|\rangle$ ensures that the Landau velocity is nowhere exceeded. Such a random walk, though clearly non-self-consistent, yields the expected length scale for spatial variations of $\Delta(\mathbf{r})$, obtained from mean-field theories.[11–13] For the simulations, we set $\phi_1^{(m)}=0.05$, $\langle|\Delta|\rangle=0.1$, and $\delta U=0.2$.

The junctions at $x=0$ and $x=L$ each have the topology of a $M+1$ pointed star and various suggestions for their scattering properties have been discussed.[18,19] In what follows, a symmetric junction scattering matrix is employed, with a normal reflection amplitude for each lead equal to $(1-M)/(1+M)$, normal transmission amplitudes from a given lead into any other equal to $2/(1+M)$, and vanishing amplitudes for Andreev scattering. By matching plane wave solutions at the boundaries of neighboring cells, the reflection and transmission coefficients for a chain of N cells can be obtained numerically from the product of $N+1$ transfer matrices. Typical results for the case of a single chain $M=1$ are presented in Fig. 2, where the left-hand graphs show how the average scattering coefficients, obtained by ensemble averaging over 1000 systems, vary with the number of cells N of the superconductor. For large N, the average transmission coefficients decay exponentially with N, while the reflection coefficients tend to limiting values whose sum is unity. In this limit, where boundary resistances R_B and R_B' are statistically independent, the simplified formula (3) gives identical results to the full form (2). From the left-hand plots of Fig. 2, it is evident that the reflection coefficients are well behaved statistically. In marked contrast, as shown by the right-hand figure, corresponding results for the average value of R show large fluctuations, which increase with system size.

For large N, as further cells are added to the right, the left boundary resistance R_B of a given sample approaches a constant value, while the right boundary resistance R_B' fluctuates. R is the sum of these two quantities, so the former defines a sample-dependent lower limit on R, while the latter yields the fluctuations. Despite the fact that this is a boundary resistance, the results of Fig. 2 suggest that as $N\to\infty$, the moments of R do not approach a limit. This is illustrated in Fig. 3, which shows numerical results for the distribution of R_B and R_B' obtained for the above model and also for an example in which the inverse localization length $\alpha=0$, obtained by setting $\delta U=0$, but keeping all other parameters the same as in Fig. 2. These results, which are typical of those found for a range of values of $\langle|\Delta|\rangle$, δU, and ϕ_1, clearly demonstrate the existence of an R^{-2} tail, and since the right-hand side of (5) diverges when $\alpha=0$, the tail is more general than the perturbative argument used to predict it. It should be noted that for our model, R_B and R_B' have different distributions, since we have forced $\Delta(x)$ to increase slowly from zero at the left boundary, while at the right boundary, it will drop sharply to zero from whatever value it has in cell N. The inset of the figure shows the distributions scaled so that they fall onto the same curve at large resistances, while keeping the total probability unity and emphasizes that the distributions approach a universal form independent of the disorder and the nature of the boundary for large R_B. It is remarkable that

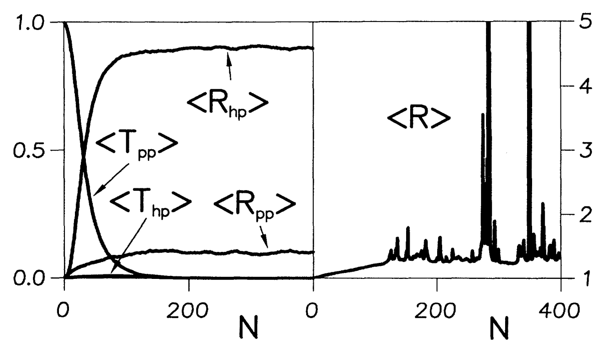

FIG. 2. For a single superconducting chain, the left figure shows numerical results for normal (R_{pp},T_{pp}) and Andreev (R_{hp},T_{hp}) reflection and transmission coefficients, as a function of the number of cells N, obtained by averaging over 1000 samples, with $\langle|\Delta|\rangle=0.1$, $\phi_1^1=0.05$, $\delta U=0.2$, and $E=10^{-3}$. The right figure shows corresponding results for the 2-probe resistance R as a function of N.

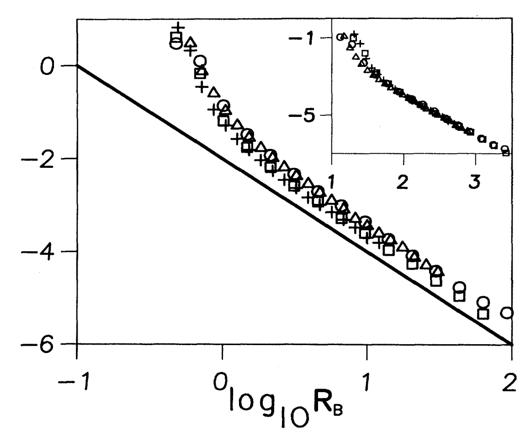

FIG. 3. Numerical results for the distribution of boundary resistances, R_B (circles) and R_B' (squares), for the same parameters as Fig. 2, with $N=400$. The triangles and crosses show results for R_B and R_B', respectively, for the same parameters except with $\delta U=0$. The solid line has a slope of -2. Th emphasize that the power-law tail is a universal feature, the inset shows the results scaled so that they lie on top of each other for large R. To achieve this scaling, the probability distributions and resistances are multiplied by a scale factor and the scale factor for each of the four distributions chosen to render the tails coincident.

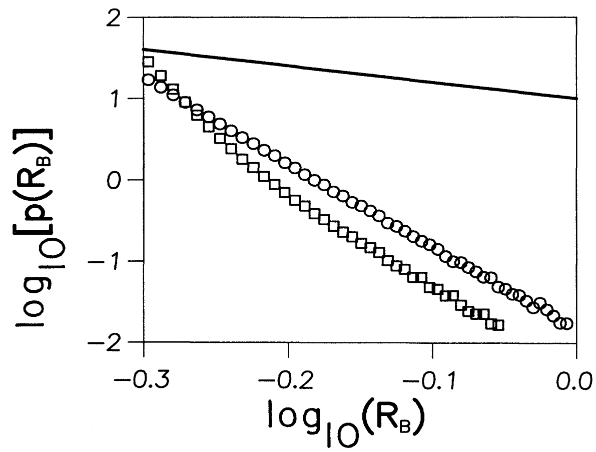

FIG. 4. The solid circles and solid squares show numerical results for the distribution of boundary resistances, for chains of length $N=400$, R_B and R_B', respectively, for the same parameters as Figs. 2 and 3, with diagonal disorder, but with $arg\Delta=0$ everywhere. A comparison with the results of Fig. 3 (shown as open squares, circles, triangles, and pluses) reveals that the $1/R^2$ tail is no longer present.

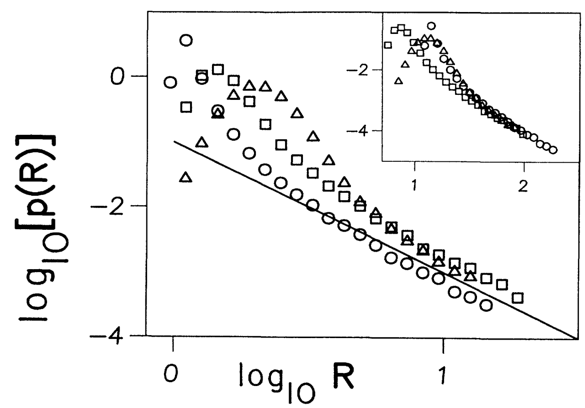

FIG. 5. Numerical results for the probability densities of resistances R. Circles are for a single wire, squares for two wires in parallel and triangles for three wires in parallel. Parameters are as for Fig. 2. The solid line has a slope of -2. The inset shows the results scaled to lie on top of each other for large R.

the R^{-2} tail remains even in the absence of diagonal disorder.

To demonstrate the importance of phase fluctuations, the solid circles and solid squares of Fig. 4 show results obtained with $y_n^1=\phi_1^1=0$ and all other parameters, as in Fig. 2. For comparison, the results of Fig. 3 are also plotted. This demonstrates that in the absence of phase fluctuations, the $1/R^2$ tail is no longer present and the average resistance is well behaved. For the same parameters used in Fig. 2, the probability densities $p(R)$ for the cases $M=1,2$, and 3 are shown in Fig. 5. It should be noted that the peaks in the distributions move to higher resistances with increasing M. This occurs because the reflection coefficient for a $M+1$ pointed star varies from 0 at $M=1$ to 1 at $M=\infty$. These results clearly demonstrate that the power-law tails survive in the presence of many parallel conducting channels and, therefore, should be observable in actual devices. For simplicity, the analysis presented above has focused on a disordered multichannel superconductor connected to one-dimensional external leads; the case of finite dimensional external leads remains to be investigated. Nevertheless, the fact that the power-law tail survives for $M>1$ suggests that the effect may be present for superconducting samples in two and three dimensions.

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