PHYSICAL REVIEW D VOLUME 24, NUMBER 9 1 NOVEMBER 1981

Interactions of high-energy ($E > 5 \times 10^{19}$ eV) photons in the Earth's magnetic field

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(Received 8 June 1981)

Cosmic-ray photons of energy $>5 \times 10^{19}$ eV will pair produce in the Earth's magnetic field. The electrons radiate by quantum synchrotron radiation; the produced photons may in turn pair produce and continue the cascade. This electromagnetic cascade interacts with the Earth's atmosphere and produces an extensive air shower. Some implications for determining the photon contribution to the highest-energy cosmic rays are discussed.

If high-energy cosmic rays are protons which fill the universe uniformly one expects a pronounced steepening of the spectrum above 5×10^{19} eV, due to photopion production.[1–3] Similarly, photodisintegration of heavy nuclei will lead to a cutoff in the spectrum at energies of $\sim 10^{19}$ eV. Contrary to predictions the observed cosmic-ray spectrum[4] continues to energies $>10^{20}$ eV and flattens above 10^{19} eV. As a result, the hypothesis that ultrahigh-energy cosmic rays (UHCR's) are local in origin has been considered.[4,5] For sources closer than 5×10^{26} cm, it is noted[6] that photon-photon pair-production interactions do not eliminate the possibility of a γ-ray component to the spectrum of particles incident on the Earth. Other work,[7–9] based on the assumption that UHCR's are of universal origin, has shown that interactions of cosmic-ray protons with the 2.7-K radiation can lead to the transference of a sizable fraction of the primary energy to photons.

The energy of a primary cosmic ray is obtained from measurements with an air-shower array and a knowledge of the development of the resulting air shower.[10,11] Such estimates assume that the air shower originates from a single particle incident on the top of the atmosphere. However, for primary photons of the highest energies, we now examine the possibility that this is not the case.

Magnetic pair production has been shown[12] to be important in the intense magnetospheres of neutron stars. Similarly[13] the path length of photons of energy 10^{25} eV traversing a cosmic magnetic field of 10^{-6} G is much less than 10^{19} cm. In what follows it is demonstrated that for photon energies $>5 \times 10^{19}$ eV, pair production within the geomagnetic field becomes an important effect and the electromagnetic cascade must traverse the equivalent of several radiation lengths before interacting with the Earth's atmosphere.

Following Erber,[14] we note that the probability of electron-positron pair production by a photon of energy $h\nu$ traversing a magnetic field of perpendicular component H is essentially governed by the parameter χ, given by $\chi(r) = \frac{1}{2}(h\nu/mc^2)(H/H_c)$. In this equation, $H_c = 4.41 \times 10^{13}$ G is the natural quantum-mechanical measure of magnetic field. The photon attenuation coefficient $\alpha(\chi)$ is related to χ by the equation

$$\alpha(\chi) = \frac{1}{2} \times \frac{1}{137} \times \frac{1}{\lambda\!\!\!^{-}_c} \left[\frac{H}{H_c} \right] T(\chi) \; , \tag{1}$$

where the dimensionless function $T(\chi)$ involves integrals over modified Bessel functions and $\lambda\!\!\!^{-}_c$ is the Compton wavelength of an electron.

To examine the possibility of pair production in the Earth's magnetic field, consider a photon incident in the plane of the magnetic equator, where $H \sim 1/r^3$ and takes a value 0.35 G at the Earth's surface. The probability that a photon initially at infinity will pair produce before reaching a position r is

$$I(r) = 1 - \exp \int_{-\infty}^{r} \alpha(\chi(r'))\, dr' \; , \tag{2}$$

while the probability that such a photon will pair produce in the interval r to $r + dr$ is $F(r)dr$, where $F(r) = \alpha(\chi)[1 - I(r)]$.

The integral may be evaluated numerically and taking 1 Earth radius $= 6.4 \times 10^8$ cm as a unit of length, the probability that a photon of energy $h\nu$ will pair produce before reaching the Earth is just $I(1)$. This quantity is given in Table I. In Table II is given the most probable position of pair production R_p, corre-

TABLE I. The magnetic pair-production probability $I(1)$, as a function of the photon energy $h\nu$.

$h\nu$ (10^{19} eV)	$I(1)$ (%)
20	92
10	59
9	50
8	41
5	10
3	1

TABLE II. The most probable position for pair production R_p in units of Earth radii, as a function of the photon energy $h\nu$.

$h\nu$ (eV)	R_p
10^{20}	1.03
10^{21}	1.62
10^{22}	2.55

sponding to the maximum of the function $F(r)$. These tables demonstrate that pair production becomes important for primary-photon energies of $h\nu \gtrsim 5\times10^{19}$ eV.

The electrons produced by such an interaction will emit photons by quantum synchrotron radiation and thus an initial photon will be rapidly degraded to a shower of photons. Indeed, these photons may themselves pair produce leading to the development of a photon-electron cascade before the atmosphere is encountered.

To examine the spectrum of magnetic bremsstrahlung, it is noted[13,15,16] that the transition probabilities for a particle of energy E in a uniform magnetic field H are again governed by the parameter $(E/mc^2)(H/H_c)$. In Ref. 13, this quantity is denoted by $Z/3$ and for a given value of Z, we denote the energy emitted/unit length into the range $h\nu$ to $h\nu+d(h\nu)$ by $J(h\nu,Z)d(h\nu)$. Denoting the peak of the emitted-photon spectrum by $\nu_{\max}$ and the total intensity of radiation emitted/unit length by J_Z, the most probable photon energies may be obtained by replacing $J(h\nu,Z)$ by $J_Z\delta(h\nu-h\nu_{\max})$.

For the situation under consideration, both Z and $\nu_{\max}$ are functions of the position r, and the total energy emitted between r_1 and r_2 by a particle traveling towards the origin is

$$E_{r_1,r_2}=\int_{r_1}^{r_2} J_Z(-dr) \quad . \tag{3}$$

The most probable bremsstrahlung spectrum is obtained from the fact that this energy is emitted in the form of discrete quanta. For example, for fixed r_1 the most probable energy $h\nu_{\max}(r_2)$ and position of emission r_2 of the first photon is the solution of the equation

$$h\nu_{\max}(r_2)=\int_{r_1}^{r_2}(-dr)J_Z(r) \quad , \tag{4}$$

where the position dependence of $\nu_{\max}$ and Z has been shown explicitly. The energy of the radiating particle at r_2 is now taken to be $E-h\nu_{\max}(r_2)$. The most probable energy and position of the next emitted photon is then calculated and so on until the electron reaches a position $r=1$. The asymptotic formulas available for J_Z in the large- and small-Z limits[14] are not applicable in the region of interest. Similarly, the low-frequency approximation discussed by Erber[14] in inapplicable and again the integrals were evaluated numerically.

Figure 1 shows the spectrum arising from a single electron of energy 5×10^{19} eV produced at a position $R_p=1.03$ Earth radii. In this case, it can be seen that an incident primary photon of energy 10^{20} eV results in a shower which consists of 2 electrons and about 40 photons. The electron energy at $r=1$ is 6×10^{18} eV, while the highest energy of a photon is 1.4×10^{19} eV. From Table I, it is noted that secondary pair production is unlikely at these energies.

To study this further possible development of the shower, electrons of higher initial energies must be considered. Table III shows the most probable energies and positions of production of the highest-energy photons which are emitted by the electrons arising from 10^{21}-eV and 10^{22}-eV primaries. These photons correspond to the $I(1)\gtrsim50\%$ threshold for further pair production.

The predicted effect of this cascading on the apparent primary cosmic-ray spectrum, obtained from analyzing the air-shower data, is dependent on the spread of the cascade before encountering the atmosphere. To our knowledge, there are no presently accepted theoretical results incorporating radiation reaction effects, which are applicable in the ultrarelativistic quantum region of interest. However, it is noted that the quasiclassical results of Shen[17] and of Erber[18] would suggest typical spreads of the order of 10^{-1} cm. Hence the air showers resulting from the individual components of the initial cascade would superimpose to yield a single shower on the ground. The rate of development of electromagnetic showers of the highest energies in the atmosphere is slowed by the Landau-Pomeranchuk density effect.[19,20] The rate of

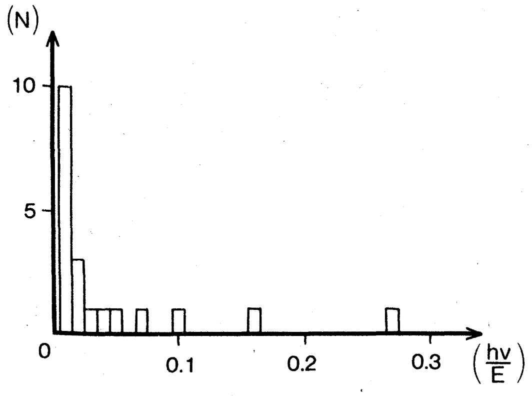

FIG. 1. The number (N) of photons emitted by a single $E=5\times10^{19}$ eV electron produced at $R_p=1.03$, as a function of the photon energy $(h\nu)$.

TABLE III. The highest emitted photon energies $h\nu$ and their most probable positions of emission R_γ in units of Earth radii, for two different electron energies E.

E (eV)	$h\nu$ (10^{20} eV)	R_γ
5×10^{20}	2.2	1.60
	0.9	1.59
5×10^{21}	34.0	2.43
	7.4	2.36
	3.0	2.32
	1.6	2.29
	0.96	2.26

TABLE IV. The incident photon energies E_{10}, E_{50}, and E_{90} in units of 10^{20} eV corresponding to the probabilities of pair production of 10%, 50%, and 90%, respectively, for three different geomagnetic latitudes θ.

θ	E_{10}	E_{50}	E_{90}
0°	0.5	0.9	1.9
30°	0.6	1.1	2.4
60°	1.2	2.5	8.0

all electromagnetic interactions is significantly reduced when the primary energy exceeds 10^{21} eV, thus slowing the initial growth of the electromagnetic cascade.[21] At these energies, for almost all geomagnetic latitudes, the electromagnetic cascade will have already commenced in the geomagnetic field thus reducing the energies of the primaries that reach the atmosphere and hence the energy-dependent density effect. At the threshold for magnetic pair production there will be significant differences in the development of the electromagnetic cascade between the photons that have and have not pair produced. These differences should be reflected in the apparent slope of the primary cosmic-ray spectrum and will depend on the geomagnetic latitude. Table IV shows the energies E_{10}, E_{50}, and E_{90} which correspond to probabilities of pair production of 10%, 50%, and 90% for primary photons incident along magnetic latitudes of $\theta = 0°$, 30°, and 60°. If a sizable fraction of the UHCR's are photons, the variation with θ of the threshold energy for pair production and the accompanying change in the primary spectrum should indeed be measurable.

It is interesting to note that for primary photons traversing a solar magnetic field of 10^{-4} G, the 10% cuton threshold for pair production is 6×10^{22} eV. The quasiclassical results[17,18] yield typical spreads for the emitted photons of 10 m at the Earth. If future estimates or measurements show the angular spread of the emitted photons to be underestimated by orders of magnitude, the air showers at the Earth resulting from the cascade components in the solar magnetic field will appear as distinct events at energies less than 10^{22} eV.

This work was sponsored in part by the National Board for Science and Technology. The work of C.J.L. was supported in part by the Department of Education.

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