Weak-Localization Magnetoresistance and Valley Symmetry in Graphene

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(Received 2 April 2006; published 5 October 2006)

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DOI: 10.1103/PhysRevLett.97.146805

PACS numbers: 73.63.Bd, 71.70.Di, 73.43.Cd, 81.05.Uw

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dominate the elastic scattering rate, \( \tau^{-1} = \tau_0^{-1} = \pi \gamma u^2 / h \), where \( \gamma = p_F / (2 \pi \hbar^2 v) \) is the density of states of quasiparticles per spin in one valley. All other types of disorder which originate from atomically sharp defects [15] and break the SU\(^2\) pseudospin symmetry are included in a time-inversion-symmetric \(^{14}\) random matrix \( \Sigma_s \Lambda_u \Lambda_v (\mathbf{r}) \). Here, \( u_{s, \mathbf{r}} (\mathbf{r}) \) describes different on-site energies on the A and B sublattices. Terms with \( u_{s, \mathbf{r}} (\mathbf{r}) \) and \( u_{v, \mathbf{r}} (\mathbf{r}) \) take into account fluctuations of \( A \equiv B \) hopping whereas \( u_{s, \mathbf{r}} (\mathbf{r}) \) and \( u_{s, \mathbf{r}} (\mathbf{r}) \) generate intervalley scattering. In addition, warping term \( \hat{h}_w \) not only breaks p → −p symmetry of the Fermi lines within each valley but also partially lifts SU\(^1\) symmetry.

Hidden SU\(^1\) symmetry of the dominant part of \( \hat{H} \) in Eq. (4) enables us to classify the two-particle correlation functions, “Cooperons” which determine the interference correction to the conductivity, \( \delta g \) by pseudospin. Below, we show that \( \delta g \) is determined by the interplay of one pseudospin singlet (C\(^0\)) and three triplet (C\(^{s,v,y}\)) Cooperons, \( \delta g \propto -C^0 + C^s + C^v + C^y \), some of which are suppressed due to a lower symmetry of the Hamiltonian in real graphene structures. That is, the “warping” term \( \hat{h}_w \) and the disorder \( \Sigma_s \Lambda_u \Lambda_v \) suppress intravalley Cooperons \( C^v,y \) and wash out the Berry phase effect and WAL whereas intervalley disorder \( \Sigma_s \Lambda_{s,1}(\mathbf{r}) \) suppresses \( C^s \) and restores weak localization [9] of electrons, provided that their phase coherence is long. This results in a WL-type negative weak-field magnetoresistance in graphene, that is absent when the intervalley scattering time is long, as we discuss at the end of this Letter.

To describe quantum transport of 2D electrons in graphene we (a) evaluate the disorder-averaged one-particle Green functions, vertex corrections, Drude conductivity, and transport time; (b) classify Cooperon modes and derive equations for those which are gapless in the limit of purely potential disorder; (c) analyze “Hikami boxes” [9,10] for the weak localization diagrams paying attention to a peculiar form of the current operator for Dirac electrons and evaluate the interference correction to conductivity leading to the WL magnetoresistance. In these calculations, we treat trigonal warping \( \hat{h}_w \) in the free-electron Hamiltonian Eqs. (1) and (4) perturbatively, assume that dominant disorder \( \hat{h}_u (\mathbf{r}) \) dominates in the elastic scattering rate, \( \tau^{-1} = \tau_0^{-1} = \pi \gamma u^2 / h \), and take into account all other types of disorder when we determine the relaxation spectra of low-gap Cooperons.

(a) Standard methods of the diagrammatic technique for disordered systems [9,10] at \( p_F v \tau \gg h \) yield the disorder-averaged single particle Green’s function,

\[
\hat{G}^{R,A}(\mathbf{p}, \epsilon) = \frac{\epsilon_{R,A} + i \Sigma \mathbf{p}}{\epsilon_{R,A} - i \Sigma \mathbf{p}}, \quad \epsilon_{R,A} = \epsilon \mp \frac{1}{2} i \hbar \tau_0^{-1}.
\]

The current operator, \( \hat{\psi} = v \Sigma \), for the Dirac-type particles described in Eq. (1) is momentum independent. As a result, the current vertex \( \hat{\psi}_j (j = x, y) \), which enters the

Drude conductivity, Fig. 1(a),

\[
g_{jj} = \frac{e^2}{\pi \hbar} \int \frac{d^2 \mathbf{p}}{(2\pi)^2} \text{Tr} \{ \hat{\psi}_j \hat{G}^R(\mathbf{p}, \epsilon) \hat{\psi}_j \hat{G}^A(\mathbf{p}, \epsilon) \},
\]

\[= 4e^2 \gamma D, \quad \text{with} \quad D = v^2 \tau_0 \equiv \frac{1}{2} v^2 \tau_w. \quad (5)
\]

is renormalized by vertex corrections in Fig. 1(b): \( \hat{\psi} = 2\hat{\psi} = 2v \hat{\Sigma} \). Here, Tr stands for the trace over the \( AB \) and valley indices. The transport time in graphene is twice the scattering time, \( \tau_0 = 2 \tau_0 \), due to the scattering anisotropy (lack of backscattering off a potential scatterer). This follows from the Einstein relation Eq. (5) (where spin degeneracy has been taken into account).

(b) The WL correction to the conductivity is associated with the disorder-averaged two-particle correlation function \( C_{\alpha \beta, \alpha' \beta'}^{\xi \mu, \xi' \mu'} \) known as the Cooperon. It obeys the Bethe-Salpeter equation represented diagrammatically in Fig. 1(c). The shaded blocks in Fig. 1(c) are infinite series of ladder diagrams, while the dashed lines represent the correlator of the disorder in Eq. (4). Here, the valley indices \( \{K_\pm\} \) of the Dirac-type electron are included as superscripts with incoming \( \xi \mu \) and outgoing \( \xi' \mu' \), and the sublattice \( (AB) \) indices as subscripts \( \alpha \beta \) and \( \alpha' \beta' \).

It is convenient to classify Cooperons in graphene as iso- and pseudospin singlets and triplets,

\[
C_{s_1 s_2} = \frac{1}{4} \sum_{\alpha \beta, \alpha' \beta'} (\Sigma_s \Lambda_s \Lambda_v)_{\alpha \beta}^{\xi \mu} (\Sigma_s \Lambda_s \Lambda_v)_{\alpha' \beta'}^{\xi' \mu'},
\]

\[\times C_{\alpha \beta, \alpha' \beta'}^{\xi \mu, \xi' \mu'}(\Sigma_s \Lambda_s \Lambda_v)_{\alpha \beta}^{\xi \mu} (\Sigma_s \Lambda_s \Lambda_v)_{\alpha' \beta'}^{\xi' \mu'}. \quad (6)
\]

Such a classification of modes is permitted by the commutation of the iso- and pseudospin operators \( \hat{\Sigma} \) and \( \hat{\Lambda} \) in

FIG. 1. (a) Diagram for the Drude conductivity with (b) the vertex correction. (c) Bethe-Salpeter equation for the Cooperon propagator with valley indices \( \xi \mu, \xi' \mu' \) and \( AB \) lattice indices \( \alpha \beta, \alpha' \beta' \). (d) “Hikami box” relating the conductivity correction to the Cooperon propagator with (e) and (f) dressed “Hikami boxes.” Solid lines represent disorder averaged \( G^{R,A} \); dashed lines represent disorder.
Eqs. (2), (3), and (6), $[\Sigma, \Lambda] = 0$. To select the isospin-singlet ($s = 0$) and triplet ($s = x, y, z$) Cooperon components (scalar and vector representation of the group SU(2) = \{e^{i\alpha\vec{\Sigma}}\}), we project the incoming and outgoing Cooperon indices onto matrices $\Sigma_x, \Sigma_y, \Sigma_z$, and $\Sigma_{x',y',z'}$, respectively. The pseudospin-singlet ($l = 0$) and triplet ($l = x, y, z$) Cooperons (scalar and vector representation of the “valley” group SU(2) = \{e^{i\beta\rho\vec{\Sigma}}\}) are determined by the projection of $C^{\mu\nu}_{\alpha\beta\gamma\delta\rho\sigma}$ onto matrices $\Lambda_y\Lambda_z \Lambda_{y'} \Lambda_{z'}$, and are accounted for by superscript indices in $C^{\mu\nu}_{\alpha\beta\gamma\delta\rho\sigma}$.

For disorder $\hat{I}u(r)$, the equation in Fig. 1(c) is

$$C_{l_z}^{l_z}(q) = \tau_0 \delta_{l_z} \delta_{s_z} + \frac{1}{4\pi g \tau_0 \hbar} \sum_{l, s_z} C_{l_s}^{l_s}(q) \int \frac{d^2 p}{(2\pi)^2} \text{Tr} \{ \Sigma_x \Sigma_y \Sigma_z \Sigma_{x'} \Sigma_{y'} \Sigma_{z'} \Sigma_{x'y'z'} \Sigma_{l_q-l_p} \}.$$

As $\hat{h}_w$ has a similar effect, it suppresses the pseudospin-triplet intravalley components $C_{l}^{l}$ and $C_{l_z}^{l_z}$, at the rate

$$\tau_w^{-1} = 2\tau_0 (e^2/\mu/h^2)^2.$$

However, since warping has an opposite effect on valleys $K_x$ and $K_y$, it does not cause gaps in the intervalley Cooperons $C_{l}^{l}$ (the only true gapless Cooperon mode) and $C_{l_z}^{l_z}$.

Alternatively, the relaxation of modes $C_{l}^{l}$ can be described by the following combinations of rates:

$$\Gamma_0 = 0, \quad \Gamma_0 = 2\tau_i^{-1}, \quad \Gamma_0 = \Gamma_0 = \tau_w^{-1} + \tau_z^{-1} + \tau_l^{-1} = \tau_*^{-1}.$$

In the presence of an external magnetic field, $B = \text{rot}\bar{A}$, and inelastic decoherence, $\tau_{\phi}^{-1}$, equations for $C_{l}^{l}$ read

$$[D(i\nabla + 2e/c\bar{A})^2 + \Gamma_0 + \tau_{\phi}^{-1} - i\omega]C_{l}^{l}(r, r') = \delta(r - r').$$

(c) Because of the momentum-independent form of the current operator $\bar{v} = 2\nu\bar{\Sigma}$, the WL correction to conductivity $\delta g$ includes two additional diagrams, Fig. 1(e) and 1(f) besides the standard diagram shown in Fig. 1(d). Each of the diagrams in Fig. 1(e) and 1(f) (not included in the analysis in Ref. [11]) produces a contribution equal to $(-\frac{1}{2})$ of that in Fig. 1(d). This partial cancellation, together with a factor of 4 from the vertex corrections and a factor of 2 from spin degeneracy, leads to

$$\delta g = \frac{2e^2D}{\pi\hbar} \int \frac{d^2 q}{(2\pi)^2} (C_{l'}^{y} + C_{l_z}^{y} + C_{l'}^{z} - C_{l_z}^{z}).$$

Using Eq. (9), we find the $B = 0$ temperature dependent correction, $\delta\rho$, to the graphene sheet resistance,

$$\frac{\delta\rho(0)}{\rho^*} = -\delta g = \frac{e^2}{\pi\hbar} \ln \left( 1 + \frac{\tau_{\phi}}{\tau_0} \right) - 2\ln \frac{\tau_{\phi}/\tau_0}{1 + \tau_{\phi}/\tau_0}.$$

and evaluate magnetoresistance, $\rho(B) - \rho(0) \equiv \Delta\rho(B)$, $\Delta\rho(B) = -\frac{\rho^*}{\pi\hbar} \left[ F(B/B_{\|}) - F(B_{\|}) - 2F(B_{\|}) \right]$, $F(z) = \ln z + \psi \left( \frac{1}{2} + \frac{1}{z} \right)$, $B_{\|,\perp} = \frac{\hbar c}{4De} \tau_\phi$, $\tau_\phi = \frac{\hbar c}{4De} \tau_\phi$. (11)

Here, $\psi$ is the digamma function, and the decoherence $\tau_{\phi}^{-1}(T)$ determines the curvature of $\Delta\rho(B)$ at $B \approx B_{\|}$.
Equations (10) and (11) represent the main result of this Letter. They show that in graphene samples with the intervalley time shorter than the decoherence time, $\tau_\varphi > \tau_i$, the quantum correction to the conductivity has the WL sign. Such behavior is expected in graphene tightly coupled to the substrate (which generates atomically sharp scatterers). Figure 2 illustrates the corresponding MR in two regimes: $B_i \sim B_j$ ($\tau_\varphi > \tau_i$) and $B_i \gg B_j$ ($\tau_\varphi < \tau_i$). In both cases, the low-field MR at $B < B_i$, is negative (for $B_i \sim B_j$, the MR changes sign at $B \sim B_j$). A dashed line shows what one would get upon neglecting the effect of warping: the solid curve shows the MR behavior in graphene with a high carrier density, where the effect of warping is strong and leads to a fast relaxation of intravalley Cooperons, at the rate described in Eq. (10). Then, in Eqs. (10) and (11) $\tau_\varphi = \tau_\psi < \tau_i < \tau_\varphi$ and $B_i < B_j$, which determines MR of a distinctly WL type. Note that in the latter case, MR is saturated at $B \sim B_j$, in contrast to the WL MR in conventional electron systems, where the logarithmic field dependence extends into the field range of $\hbar c/4D\tau_i$. In a sheet loosely attached to a substrate (or suspended), the intervalley scattering time may be longer than the decoherence time, $\tau_i > \tau_\varphi > \tau_w$ ($B_i < B_\varphi < B_j$). In this case, $C_0^\varphi$ in Eq. (11) is effectively gapless and cancels $C_0^\psi$, whereas trigonal warping suppresses the modes $C_0$ and $C_0^\varphi$, so that $\delta g = 0$ and MR displays neither WL nor WAL behavior: $\Delta \rho(B) = 0$.

Equation (11) explains why in the recent experiments on quantum transport in graphene [17] the observed low-field MR displayed a suppressed WL behavior rather than WAL. For all electron densities in the samples studied in [17] the estimated warping-induced relaxation time is rather short, $\tau_\psi/\tau_w \sim 5-30$, $\tau_\varphi < \tau_\varphi$, which excluded any WAL. Moreover, the observation [17] of a suppressed WL MR in devices with a tighter coupling to the substrate agrees with the behavior expected in the case of sufficient intervalley scattering, $\tau_i < \tau_\varphi$, whereas the absence of any WL MR, $\Delta \rho(B) = 0$, for a loosely coupled graphene sheet is what we predict for samples with a long intervalley scattering time, $\tau_i > \tau_\varphi$.

In a narrow wire with the transverse diffusion time $L_\perp^2/D \ll \tau_i$, $\tau_i \geq \tau_\varphi$ edges scatter between valleys [18]. Thus, we estimate $\Gamma_0 \sim \pi^2 D/L_\perp^2$ for the pseudospin triplet in a wire, whereas the singlet $C_0^\varphi$ remains gapless. This yields negative MR for $B \leq 2\pi B_\perp$, $B_\perp = \hbar c/eL_\perp^2$:

$$\frac{\Delta \rho\text{(wire)}(B)}{\rho_0} = \frac{2e^2L_\perp^2}{\hbar} \frac{1}{\sqrt{1 + \frac{1}{3} B^2/B_\perp^2 B_\perp^2}} - 1.$$  (12)

Equations (10)–(12) completely describe the WL effect in graphene and explain how the WL magnetoresistance reflects the degree of valley symmetry breaking. They show that, despite the chiral nature of electrons in graphene suggestive of antilocalization, their long-range propagation in a real disordered material or a narrow wire does not manifest the chirality.

We thank I. Aleiner, V. Cheianov, A. Geim, P. Kim, O. Kashuba, and C. Marcus for discussions. This project has been funded by the EPSRC Grant No. EP/C511743.

References:


[8] Here, $K_+ = \pm (\hbar^2 a^{-1})$, $a$ is the lattice constant.


[13] The group $U_4$ can be described using 16 generators $I$, $\Sigma_i$, $\Lambda_i$, $\Lambda_i^*$; $I = x, y, z$.

[14] In the basis $\Phi = \{\phi_{K,\pm}, \phi_{K,0}, \phi_{K,0}\}$, time reversal of an operator $W$ is $T(W) = (\Pi_+ \Phi \sigma_3)^W (\Pi_+ \Phi \sigma_3)$, and $T(\Sigma_i) = -\Sigma_i$, $T(\Lambda_i) = -\Lambda_i$, $T(\Lambda_i^*) = -\Lambda_i^*$.


