A tunnel junction between a ferromagnet and a normal metal: magnon-assisted contribution to thermopower and conductance

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Abstract

We develop a theoretical model of magnon-assisted transport in a mesoscopic tunnel junction between a ferromagnetic metal and a normal (nonmagnetic) metal. The current response to a bias voltage is dominated by the contribution of elastic processes rather than magnon-assisted processes and the degree of spin polarization of the current, parameterized by a function $P(P_m(k), P_N)$, depends on the relative sizes of the majority $P_m(k)$ and minority $P_N$ band Fermi surface in the ferromagnet and of the Fermi surface of the normal metal $P_N$. On the other hand, magnon-assisted tunneling gives the dominant contribution to the current response to a temperature difference across the junction. The resulting thermopower is large, $S \sim -(k_B/e)(k_B T/\omega_D)^{3/2} P(P_m(k), P_N)$, where the temperature dependent factor $(k_B T/\omega_D)^{3/2}$ reflects the fractional change in the net magnetization of the ferromagnet due to thermal magnons at temperature $T$ (Bloch’s $T^3$ law) and $\omega_D$ is the magnon Debye energy.

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1. Introduction

For more than a decade there has been intensive research of spin polarized transport [1] with a view to exploit phenomena such as the tunneling magnetoresistance (TMR) of ferromagnetic junctions [2,3] and the giant magnetoresistance (GMR) in multilayer structures [4,5]. The TMR effect arises because the mismatch of spin currents at the interface between two ferromagnetic (F) electrodes with antiparallel spin polarizations produces a larger contact resistance than a junction with parallel polarizations. In general, the magnitude of TMR increases with the degree of polarization of the ferromagnets, but it is reduced by spin relaxation phenomena [1,6], such as spin–orbit scattering at impurities or magnon emission, which lift spin current mismatch. Magnon emission is an inelastic process that has been studied both theoretically [7] and experimentally [8] in...
ferromagnetic tunnel junctions with a view to relate nonlinear $I(V)$ characteristics to the density of states of magnons $\Omega(k)$ as $d^2I/dV^2 \propto \Omega(eV)$. As well as influencing the magnetoresistance, magnon emission is expected to produce a magnetothermopower effect: a larger thermopower in the anti-parallel orientation than in the parallel one [9,10].

Meanwhile, interest in transport across junctions between ferromagnets and normal metals or semiconductors has focused on the possibility of injecting spin into the normal system [6,11,12]. Current in the ferromagnet (F) is carried unequally by majority and minority carriers so that a current flowing across the interface with a normal conductor is expected to have a finite degree of spin polarization $P$, $0 \leq P \leq 1$. Spin injection from a ferromagnetic metal into a normal metal (N) has been measured in broad agreement with theory [6,11], but there has been difficulty in achieving large degrees of spin injection into a semiconductor at room temperature [13]. A limiting factor is believed to be conductivity mismatch [14], a problem that may be overcome by introducing a tunneling barrier between the ferromagnet and semiconductor [15]. Spin injection in all-semiconducting devices has generally been more successful [16], although it is limited to relatively low temperatures at the moment.

In this paper, we investigate the opposite effect: the injection of charge caused by the equilibration of magnetization between ferromagnetic baths kept at different temperatures. The magnetization of a ferromagnet held at a finite temperature $T$ is less that its maximum value due to the thermal occupation of magnons, with the fractional change $\delta m(T)$ obeying Bloch’s $T^{3/2}$ law,

$$\delta m(T) = \frac{3.47}{\xi} \left(\frac{k_BT}{\omega_D}\right)^{3/2},$$

where $\xi$ is the spin of the localized moments and $\omega_D$ is the magnon Debye energy. Under certain conditions the transfer of heat across a junction between ferromagnets or a ferromagnet and a normal metal may involve simultaneous spin transfer and, possibly, charge transfer. Therefore, we study the influence of magnon assisted processes on the transport properties of a mesoscopic size ferromagnet/insulator/normal metal tunnel junction. The bottle-neck of both charge and heat transport lies in a small-area tunnel contact between the electrodes held at different temperatures and/or electric potentials. The main result is a large thermopower $S$ arising from the magnon-assisted processes that depends on the difference between the size of the majority and minority band Fermi surfaces in the ferromagnet,

$$S \approx -\frac{k_B}{e} \frac{\delta m(T)}{2} P(\Pi_{\uparrow}, \Pi_N).$$

This result holds in a range of temperatures given by

$$1 \gg \frac{\delta m(T)}{k_B T} / \varepsilon_F,$$

where $\varepsilon_F$ is the Fermi energy. The function $P$, $0 \leq P \leq 1$, is the degree of spin polarization of the current response to a bias voltage and it depends, in general, on the area of the maximal cross-section of the Fermi surface in the plane parallel to the interface of majority $\Pi_\uparrow$ and minority $\Pi_\downarrow$ electrons in the ferromagnet, and of the electrons in the normal (nonmagnetic) metal $\Pi_N$. As an example,

$$P = \frac{(\Pi_N - \Pi_\downarrow)}{(\Pi_N + \Pi_\downarrow)},$$

for a momentum-conserving tunneling model with $\Pi_\uparrow \geq \Pi_N > \Pi_\downarrow$.

The magnon-assisted processes that we consider are similar to those discussed in Refs. [7,9,10] in relation to transport in F–F tunnel junctions. Microscopically, a typical magnon-assisted process that contributes to the thermopower in a F–N junction, Eq. (2), is shown schematically in Fig. 1. Here, the majority electrons in the ferromagnet on the left-hand side of the junction are ‘spin-up’ while the minority electrons are ‘spin-down’. In the normal (nonmagnetic) metal on the right, the density of states of the spin-up and spin-down conduction electrons are equal. The transition begins with a spin-down electron on the right, that then tunnels through the barrier (without spin flip) into an intermediate, virtual state with spin-down minority polarization on the left (Fig. 1(a)). In the final step, Fig. 1(b), the electron emits a magnon and incorporates itself into a previously unoccupied state in the spin-up majority band on the left.
power coefficient is
\[ S = -\frac{V}{\Delta T} = \frac{L}{G} \]  
\[ (6) \]

The electrical conductance is dominated by a contribution from elastic processes \( G \approx G_{el} \) that involve tunneling from both the majority and minority bands of the ferromagnet to the conduction band of the normal metal (and vice versa) without spin flip scattering. However the contribution of the same processes to the \( \Delta T \) response is generally small, \( L_{el} \sim (k_B/e)(T/\varepsilon_F)G_{el} \), as it contains the additional parameter \( T/\varepsilon_F \). By way of contrast, we find that responses arising from magnon-assisted transport are of the same order, \( L_{in} \sim (k_B/e)G_{in} \sim (k_B/e)\delta m(T)G_{el} \), both containing the parameter \( \delta m(T) \). The result, in the temperature regime defined above in Eq. (3), is that the overall \( \Delta T \) response is dominated by inelastic processes \( L \approx L_{in} \geq L_{el} \) producing a large thermopower \( S \approx L_{in}/G_{el} \), Eq. (2).

The paper is organized as follows. In Section 2 we introduce the model and technique used for describing transport across a tunnel junction and in Section 3 we calculate the current including the contribution of elastic processes and the influence of magnon-assisted processes. Section 4 gives the resulting thermopower for two different models of the interface: a uniformly transparent interface where the component of momentum parallel to the interface is conserved, and a randomly transparent interface.

2. Description of the model

We consider a tunnel junction between a ferromagnetic metal on the left (L) and a normal (non-magnetic) metal on the right (R) with, in general, a temperature drop \( \Delta T \) and a bias voltage \( V \) across the junction. Our initial aim is to write a balance equation for the current \( I(V, \Delta T) \) in terms of the occupation numbers of electrons \( n_{L,R}(\varepsilon) = \exp((\varepsilon - \varepsilon_{F,L,R})/(k_B T_{L,R})) + 1 \)\(^{-1} \) on the left (right) hand side of the junction and of magnons \( N_L(q) = \exp(\omega_q/(k_B T_L)) - 1 \)\(^{-1} \) in the ferromagnet. Here \( T_{L,R} \) is the temperature on the left (right) hand side, \( \varepsilon_{F,L,R} \) is the Fermi energy on the left (right) hand side, and \( \omega_q \) is

In our approach, we take into account inelastic tunneling processes that involve magnon emission and absorption in the ferromagnet, as well as elastic electron transfer processes, in order to obtain a balance equation for the current \( I(V, \Delta T) \) as a function of bias voltage, \( V \), and of the temperature drop, \( \Delta T \). In the linear response regime the electrical current may be written as
\[ I = GV + L \Delta T, \]  
\[ (5) \]
the energy of a magnon of wavevector \( \mathbf{q} \). In the following we set \( T_L = T + \Delta T \) and \( T_R = T \) and we shall speak throughout in terms of the transfer of electrons with charge \(-e\). The index \( \alpha \equiv \{ \uparrow, \downarrow \} \) takes account of the splitting of conduction band electrons into ‘spin-up’ \( \epsilon_{\mathbf{k} \alpha}^{L(R)} \) and ‘spin-down’ \( \epsilon_{\mathbf{k} \uparrow}^{L(R)} \) subbands. We assume that the majority electrons in the ferromagnet are spin-up so that \( \epsilon_{\mathbf{k} \uparrow}^{L} = \epsilon_{\mathbf{k} \uparrow}^{R} - \Delta/2 \) and \( \epsilon_{\mathbf{k} \downarrow}^{L} = \epsilon_{\mathbf{k} \downarrow}^{R} + \Delta/2 \) where \( \epsilon_{\mathbf{k} \alpha} \) is the bare electron energy and \( \Delta \) is the spin splitting energy. In the nonmagnetic metal on the right, \( \epsilon_{\mathbf{k} \uparrow}^{R} = \epsilon_{\mathbf{k} \downarrow}^{R} = \epsilon_{\mathbf{k} \alpha}^{R} \).

The total Hamiltonian of the system is

\[
H = H_F^T + H_N^R + H_T,
\]

(7)

where \( H_N^R \) is the Hamiltonian describing the conduction band electrons of the nonmagnetic metal on the right written in terms of creation and annihilation Fermi operators \( c^\dagger \) and \( c \). The term \( H_T \) is the tunneling Hamiltonian \([17–19]\), \( \alpha \equiv \{ \downarrow, \uparrow \} \) and we assume that spin is conserved when an electron tunnels across the interface. The tunneling matrix elements \( t_{\mathbf{k} \mathbf{k}'} \) describe the transfer of an electron with wavevector \( \mathbf{k} \) on the left to the state with \( \mathbf{k}' \) on the right. In our model \([20,10]\) we neglect its explicit energy dependence, but describe both clean and diffusive interfaces by taking into account whether the component of momentum parallel to the interface is conserved.

The term \( H_F^T \) is the Hamiltonian of the ferromagnetic electrode on the left side of the junction in the absence of tunneling. We use the so-called s-f (s-d) model \([21,22]\), which assumes that magnetism and electrical conduction are caused by different groups of electrons that are coupled via an intra-atomic exchange interaction, although we note that the same results, in the lowest order of electron–magnon interactions, may be obtained from a model of itinerant ferromagnets \([23]\). The magnetism originates from inner atomic shells (e.g., d or f) which have unoccupied electronic orbitals and, therefore, possess magnetic moments whereas the conduction is related to electrons with delocalized wave functions. Using the Holstein–Primakoff transformation \([24]\) the operators of the localized moments in the interaction Hamiltonian can be expressed via magnon creation and annihilation operators \( b^\dagger, b \). At low temperatures, where the average number of magnons is small \( \langle b^\dagger b \rangle \ll 2 \xi \) (\( \xi \) is the spin of the localized moments), the Hamiltonian of the ferromagnet \( H_F^T \) can be written as follows:

\[
H_F^T = H_F^L + H_m^L + H_{em}^L,
\]

(10)

The first term \( H_F^L \), Eq. (11), deals with conduction band electrons which are split into majority \( \epsilon_{\mathbf{k} \uparrow}^{L} \) and minority \( \epsilon_{\mathbf{k} \downarrow}^{L} \) subbands due to the s-f (s-d) exchange. The Hamiltonian \( H_m^L \), Eq. (12), describes free magnons with spectrum \( \omega_q \) which in the general case has a gap \( \omega_{q=0} = \omega_0 \). The third term \( H_{em}^L \), Eq. (13), is the electron–magnon coupling resulting from the intra-atomic exchange interaction between the spins of the conduction electrons and the localized moments.

The calculation is performed using standard second-order perturbation theory \([25]\). We write the total Hamiltonian, Eq. (7), as \( H = H_0 + V \), where the perturbation \( V = H_T + H_{em}^L \) is the sum of the tunneling Hamiltonian and the electron–magnon interactions in the left electrode. First-order terms provide an elastic contribution to the current that do not involve any change of the spin orientation of the itinerant electrons, whilst second-order terms account for inelastic, magnon-assisted processes.
3. Current across a ferromagnetic-normal junction

3.1. Elastic contribution to the current

The first-order contribution to the current arises from elastic tunneling without any spin flip between either a spin-up majority conduction electron state on the left and a spin-up state on the right or a spin-down minority state on the left and a spin-down state on the right. Consider for example an initial state consisting of an additional majority spin-up electron on the left with wavevector \( k_L \) and energy \( \varepsilon_{k_{L \uparrow}} \). This electron can tunnel, with matrix element \( t_{k_L k_R} \), into a spin-up state on the right with wavevector \( k_R \) and energy \( \varepsilon_{k_{R \uparrow}} \). In addition there is a second process which is a transition between the same two states, but in the reverse order, giving a contribution to the current with an opposite sign. Together, the two processes give a balance equation for the first-order contribution to the current between the spin-up majority band on the left and the spin-down minority band on the left and the spin-up band on the right. Overall, the first-order contribution to the current is \( I_{el} \) where

\[
I_{el} = -4\pi^2 e^2 \hbar \int_{-\infty}^{+\infty} \frac{dx}{x} \sum_{k_L, k_R} \sum_{x=\uparrow,\downarrow} |t_{k_L k_R}|^2 \delta(x - \varepsilon_{k_{L \uparrow}}) \delta(x - eV - \varepsilon_{k_{R \uparrow}}) \\
\times \left\{ n_L(\varepsilon_{k_{L \uparrow}}) \left[ 1 - n_R(\varepsilon_{k_{R \uparrow}}) \right] \\
- \left[ 1 - n_L(\varepsilon_{k_{L \uparrow}}) \right] n_R(\varepsilon_{k_{R \uparrow}}) \right\}.
\]

(14)

Keeping only contributions linear in \( V \) or \( \Delta T \), the elastic contribution to the current may be written as

\[
I_{el} \approx \frac{e^2}{\hbar} V \left[ T_{\uparrow N}(\varepsilon_F) + T_{\downarrow N}(\varepsilon_F) \right] \\
- \frac{\pi^2 e}{3 \hbar} k_B T \frac{d}{d\varepsilon} \left[ T_{\uparrow N}(\varepsilon) + T_{\downarrow N}(\varepsilon) \right] \bigg|_{\varepsilon_F}.
\]

(15)

For convenience, we have grouped all the information about the quality of the interface into a parameter \( T_{2N} \),

\[
T_{2N}(\varepsilon) \approx 4\pi^2 \sum_{k_L, k_R} |t_{k_L k_R}|^2 \delta(\varepsilon - \varepsilon_{k_{L \uparrow}}) \delta(\varepsilon - \varepsilon_{k_{R \uparrow}}),
\]

(16)

that is equivalent to the sum over all scattering channels, from states with spin \( z \) on the left to states on the right (where both spin channels are equivalent), of the transmission eigenvalues usually introduced in the Landauer formula [26–28], although we restrict ourselves to the tunneling regime in this paper. Later we will employ models of two types of interface explicitly: a uniformly transparent interface where the component of momentum parallel to the interface is conserved, and a randomly transparent interface.

The first term in Eq. (15) accounts for the usual (large) contribution to the electrical conductance,

\[
G_{el} = \frac{e^2}{\hbar} \left[ T_{\uparrow N}(\varepsilon_F) + T_{\downarrow N}(\varepsilon_F) \right]
\]

and we define the corresponding degree of spin polarized current \( P \) as the relative difference in the current due to majority and minority carriers,

\[
P = \frac{(T_{\uparrow N} - T_{\downarrow N})}{(T_{\uparrow N} + T_{\downarrow N})}.
\]

The thermopower coefficient \( S_{el} = -V/\Delta T \) due to the elastic processes may be determined from Eq. (15) by setting \( I_{el} = 0 \),

\[
S_{el} \approx -\frac{\pi^2 k_B T}{3} \frac{d}{d\varepsilon} \left[ \ln \left( T_{\uparrow N}(\varepsilon) + T_{\downarrow N}(\varepsilon) \right) \right] \bigg|_{\varepsilon_F},
\]

(19)

which is equivalent to the Mott formula [29].

3.2. Magnon-assisted contribution to the current

Below we describe processes which contribute to magnon-assisted tunneling. We consider four processes, that are lowest order in the electron–magnon interaction, as shown schematically in Fig. 2. The straight lines show the direction of electron transfer, whereas the wavy lines denote the emission or absorption of magnons. The processes are drawn using the rule, appropriate for ferromagnetic electron–magnon exchange, that an electron in a minority state scatters into a majority state by emitting a magnon. The upper two processes in Fig. 2, (i) and (ii), involve
transitions into (from) a majority final (initial) state on the left via an intermediate, virtual state in the minority band. For example, process (i), which is the same as the process shown in more detail in Fig. 1, begins with a spin-down electron on the right with wavevector \(k_R\) and energy \(e^{R}_{k_R}\). Then, this electron tunnels across the barrier (without spin flip) to occupy a virtual, intermediate state with wavevector \(k_L\) in the spin-down minority band on the left as depicted in the left part of Fig. 1(a) with energy \(e^{L}_{k_L}\). The energy difference between the states is \(e^{L}_{k_L} - e^{R}_{k_R} = e^{L}_{k_L} - e^{R}_{k_R} + \Delta\) so that the matrix element for the transition contains an energy in the denominator related to the inverse lifetime of the electron in the virtual state. For \(k_B T, eV \ll \Delta\), when both initial and final electron states should be taken close to the Fermi level, only long wavelength magnons can be emitted, so that the energy deficit in the virtual states can be approximated as \(e^{L}_{k_L} - e^{R}_{k_R} + \Delta \approx \Delta\). As noticed in Refs. [21,22], this cancels out the large exchange parameter since the same electron–core spin exchange constant appears both in the splitting between minority and majority bands and in the electron–magnon coupling. The second part of the transition is sketched in Fig. 1(b) where the electron in the virtual minority spin-down state incorporates itself into a state in the majority spin-up band on the left, wavevector \(k'\), energy \(e^{L}_{k'1}\), by emitting a magnon of wavevector \(q\).

Similar considerations enable us to write down the contribution to the current from all the processes in Fig. 2. We group the processes into pairs which involve transitions between the same series of states, but in the opposite time order so that they give a current with different signs, hence their sum gives a balance equation. The contribution to the current of the processes (i) and (ii), labelled as \(I_1\) because the state on the right is spin-down, is given by

\[
I_1 = -4\pi^2 e \int_{-\infty}^{+\infty} \frac{d\omega}{h} \sum_{k_{k_R}q} \left| \frac{k_{k_R}^L}{2\xi N} \right|^2 \delta(e - e^L_{k_L} + \omega_q) \delta(e - e^{R}_{k_R} + \omega_q) \times \left\{ -n_R(e^{R}_{k_R}) \left[ 1 - n_L(e^{L}_{k'1}) \right] \right\} [1 + N_L(q)] + \left\{ -n_R(e^{R}_{k_R}) \right\} n_L(e^{L}_{k'1}) N_L(q).
\]

We use the definition of the tunneling parameter \(T_{2N}\), Eq. (16), in order to express the currents as

\[
I_1 = -e \frac{T_{2N}}{h} \int_{-\infty}^{+\infty} d\omega \Omega(\omega) \times \left\{ N_L(\omega) n_L(\omega) \left[ 1 - n_R(\omega + e) \right] \right\} [1 + N_L(\omega)] - \left[ 1 + N_L(\omega) \right] [1 - n_L(\omega)] n_R(\omega + e - eV),
\]

(22)
\[ I_1 = + \frac{e}{h^2} \text{Tr} \int_{-\infty}^{\infty} d\omega \int_{0}^{\infty} d\epsilon \Omega(\omega) \times \{ N_L(\omega) n_R(\epsilon - \omega - eV)[1 - n_L(\omega)] \\
- [1 + N_L(\omega)][1 - n_R(\epsilon - \omega - eV)] n_L(\epsilon) \}, \] (23)

where \( \Omega(\omega) = \sum_q \delta(\omega - \omega_q) \) is the magnon density of states in the ferromagnet. Since our main aim is to demonstrate the existence of an effect, we choose the simple example of a bulk, three-dimensional magnon density of states. We assume a quadratic magnon dispersion, \( \omega_q = Dq^2 \), and apply the Debye approximation with a maximum magnon energy \( \omega_D = D(6\pi^2/v)^{2/3} \) where \( v \) is the volume of a unit cell. This enables us to express the magnon density of states as \( \Omega(\omega) = (3/2)N\omega^{-1/2}/\omega_D^{3/2} \).

Since we are interested in the linear thermopower, we expand the electron and magnon occupation numbers and keep only terms linear in \( V \) and \( \Delta T \). The resulting expression for the currents \( I_1 \) and \( I_2 \), Eqs. (22) and (23) respectively, is

\[ I_1 = \frac{e}{h^2} \text{Tr} \int_{-\infty}^{\infty} d\omega \int_{0}^{\infty} d\epsilon \Omega(\omega) \times \{ eV \Gamma(\xi(\epsilon))(\xi(\epsilon) + \frac{k_B \Delta T \Gamma(\xi(\epsilon))}{2}) \}. \] (24)

where \( \Gamma(x) \) is the gamma function and \( \xi(x) \) is Riemann’s zeta function [30].

4. Calculation of the thermopower

The thermopower \( S \) is determined by setting the total current to zero, \( I = I_1 + I_1 + I_2 = 0 \), and finding the voltage \( V \) induced by the temperature difference \( \Delta T \), \( S = -V/\Delta T \). In the regime of temperatures given in Eq. (3) the total current may be approximated by

\[ I \approx \frac{e^2}{h} V[T_1 + T_1] \\
- \frac{3e}{h} \frac{k_B T_1}{8 \xi} \Gamma(\xi(\epsilon))(\xi(\epsilon) + \frac{k_B \Delta T}{2})^{3/2} k_B \Delta T [T_1 + T_1], \] (25)

where the leading term proportional to \( V \) arises from the elastic processes, Eq. (15), and the leading term proportional to \( \Delta T \) comes from the magnon-assisted processes, Eq. (24). The corresponding thermopower is

\[ S = -\frac{k_B}{e} \frac{\delta m(T)}{\delta m(T)} P(\Pi_1, \Pi_2). \] (26)

This is the main result of the paper, describing junctions between normal metals and ferromagnets of arbitrary polarization strength ranging from weak ferromagnets \( \Pi_1 \sim \Pi_2 \) to half-metals \( \Pi_1 \gg \Pi_2 = 0 \). The function \( \delta m(T) \) in Eq. (26) is the change in the magnetization due to thermal magnons at temperature \( T \) (Bloch’s T^{3/2} law) [31],

\[ \delta m(T) = \frac{1}{\zeta N} \int_{0}^{\infty} d\omega \Omega(\omega)N_L(\omega) \]
\[ = \Gamma(\xi(\epsilon))^{3/2} \Gamma(\xi(\epsilon))^{3/2}. \] (27)

The function \( P \) appearing in the thermopower, Eq. (26), is the degree of spin polarized current \( P \) that flows in response to a bias voltage as defined in Eq. (18). We consider two specific models of the interface: a uniformly transparent interface where the component of momentum parallel to the interface is conserved, and a randomly transparent interface. As the form of the tunneling parameter \( T_{\infty N} \), Eq. (16), has been given previously [10], we only present the results here. For a uniformly transparent interface of area \( A \), such that the parallel component of momentum is conserved upon tunneling, then

\[ T_{\infty N}^{\text{flat}} \approx 4\pi^2 |t|^2 \frac{A}{\hbar^2} \min \{ \Pi_2, \Pi_N \}, \] (28)

where \( t \) represents the transparency of the interface, \( \Pi_2 \) is the area of the maximal cross-section of the Fermi surface of spins \( x \) in the ferromagnet, \( \Pi_1 \gg \Pi_2 \gg 0 \), and \( \Pi_N \) is the area of the maximal cross-section of the Fermi surface in the normal metal. For a uniformly transparent interface,

\[ p^{\text{flat}} = \frac{\min \{ \Pi_1, \Pi_N \} - \min \{ \Pi_1, \Pi_N \}}{\min \{ \Pi_1, \Pi_N \} + \min \{ \Pi_1, \Pi_N \}}. \] (29)

As an opposite extreme, we also consider a strongly nonuniform interface which is transparent in a finite number of points only, each of a typical area \( a \approx l_z^2 \) randomly distributed over the interface area \( A \). In this case

\[ T_{\infty N}^{\text{dis}} \approx 4\pi^2 |t|^2 \frac{a}{\hbar^2} \frac{\min \{ a \Pi_2, a \Pi_N \}}{\min \{ a \Pi_1, a \Pi_N \}}, \] (30)
\[ p_{\text{dis}} = \frac{(\Pi_1 - \Pi_{\downarrow})}{(\Pi_1 + \Pi_{\downarrow})}. \]  

The large thermopower Eq. (26) indicates that the current response to \( \Delta T \) of magnon-assisted processes is very efficient, as compared to elastic processes. We view this as being due to an attempt by the bath of magnons to alter its temperature. Since the population of magnons defines the magnetization of the ferromagnet, through the function \( \delta m(T) \) Eq. (27), thermal equilibration achieved by changing the magnon population must be accompanied by a change in magnetization. This is mediated by conduction electrons, resulting in a net current response to a temperature difference across the junction Eqs. (24) and (25). For example, the process shown in Fig. 2(ii), in which an electron on the left absorbs a magnon and tunnels to the right, results in a reduction in the number of magnons and the injection of a spin-down electron to the right. On the other hand, the process shown in Fig. 2(iii), in which an electron on the right tunnels to the left and absorbs a magnon, results in a reduction in the number of magnons and the collection of a spin-up electron from the right. These two processes, while both lowering the number of magnons, give competing contributions to the current as demonstrated by the factor \( T^3 \) in Eq. (25).

5. Conclusion

The main result of this work is a large contribution to the thermopower Eq. (26) due to magnon-assisted processes in ferromagnetic-normal metal tunnel junctions. As a rough estimate, we take \( \delta m = 7.5 \times 10^{-6} \times 3/2 \) (for a ferromagnet such as Ni, Ref. [31]) and \( P \sim 0.4 \) to give \( S \sim -1 \mu V K^{-1} \) at \( T = 300 \) K. Our simple model describes tunneling between parabolic conduction bands typical of three-dimensional metallic systems, with additional spin splitting and electron–magnon interactions in the ferromagnet. However we believe the main results will have qualitative relevance for junctions with semiconducting elements, too. Recent numerical modeling of the diluted magnetic semiconductor (Ga,Mn)As using a six-band Kohn–Luttinger Hamiltonian [32] found evidence of quadratic dispersion of long-wavelength spin waves \( \omega_q = \omega_0 + Dq^2 \) with a small anisotropy gap \( \omega_0 \). A fit at small momenta to their data for typical sample parameters yields a spin waves stiffness \( D = 2 \text{meV nm}^2 \) that corresponds to \( \delta m \sim 2.5 \times 10^{-4} \times 3/2 \). If this is inserted into our formula for the thermopower Eq. (26) with \( P \sim 0.4 \), say, it gives \( |S| \sim 4 \mu V K^{-1} \) at \( T = 100 \) K. However, we stress that the sign of the thermopower Eq. (26) is specified for electron (charge \( -e \)) transfer processes between parabolic conduction bands and under the assumption that the exchange between conduction band and core electrons has a ferromagnetic sign. We considered a bulk, three-dimensional magnon density of states, but in general the magnitude and sign of the thermopower will depend on details of the magnon spectrum.

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