Spin-Orbit Coupling Effect on Quantum Hall Ferromagnets with Vanishing Zeeman Energy

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We present the phase diagram of a ferromagnetic \( \nu = 2N + 1 \) quantum Hall liquid in a narrow quantum well with vanishing single-particle Zeeman splitting, \( e_Z \), and a pronounced spin-orbit coupling. Upon decreasing \( e_Z \) the spin-polarization field of a liquid takes, first, the easy-axis configuration, followed by the formation of a helical state which affects the transport and NMR properties of a liquid and the form of topological defects in it.

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The concept of quantum Hall ferromagnets (QHF) [1–3] is now broadly accepted as an approach to treat the 2D electron systems in the vicinity of odd-integer filling factors, \( \nu = 2N + 1 \), in particular, due to the success in the theory [3,4] and experimental observations [5–7] of quantum Hall effect skyrmions, which are electrically charged topological textures in the ferromagnetic order parameter in two dimensions. The attempts to stabilize skyrmions [3,4], as compared to electrons and holes on the top of a narrow quantum well (QW) [8], as a result, a narrow quantum well is a system, where Zeeman energy can be swept through zero, and where the tendency of interacting electrons at \( \nu = (2N + 1) \) to form a ferromagnetic state (with a large exchange energy \( \mathcal{J} \gg e_Z \)) polarized along the external magnetic field confronts the effects of the SO coupling [15–17] itself, as an alternative source for choosing the spin-polarization direction for 2D electrons.

In GaAs/AlGaAs quantum wells grown along the [001] facet, the effective two-dimensional SO coupling has the form of \( H_{so} = \hat{u}_{so}(p_x\sigma^+ - p_y\sigma^-) + u_{so} e^{ijz}p_i\sigma^j \), where \( p = -i\nabla - eA/c \) is the electron momentum \( [A = (0,xB_x,0)] \), \( \sigma^j \) are Pauli matrices, and \( \hbar = 1 \). The first term in \( H_{so} \) reflects the lack of inversion symmetry in the well-grown along the [001] facet with square 2D lattice symmetry and comes from the \((\gamma p^x\sigma)\) nonparabolicity in the conduction band of bulk GaAs [11,16], so that \( \hat{u}_{so} = \gamma(p_z^2) \). Experimentally [17] and theoretically [16] found values of \( \gamma \) range from \( \gamma = 27 \) to \( 22 \) eV Å\(^3\). The second SO term is due to the quantum well potential asymmetry [15]. In a narrow QW, with \( \hat{u} = 2^{-1/2}\sin(\pi x/l) \) transverse part of the electron wave function, \( \tilde{u}_{so} = \gamma(\pi x/l)^2 \gg u_{so} [17] \). (Directions for \( x \) and \( y \) axes are chosen in such a way that \( \tilde{u}_{so}, u_{so} > 0 \)).

In the present paper, we study the effect of spin-orbit coupling on the \( \nu = (2N + 1) \) QHF formed by electrons with a vanishing single-particle Zeeman energy in a narrow QW in a perpendicular magnetic field. In this analysis, we address the properties of narrow wells, since they have a more prominent SO coupling in the 2D electron Hamiltonian. In a high magnetic field providing \( \omega_c = |eB_z|/mc \gg \tilde{u}_{so}/\lambda (\lambda = \sqrt{c/eB_z}) \), a weak SO coupling does not alter Landau quantization. However, it affects the evolution of ferromagnetic properties of a quantum Hall effect liquid upon sweeping \( e_Z \) through zero. As in ordinary ferromagnets, SO coupling results in the crystalline anisotropy field [18], that deflects spin polarization \( \textbf{n} \) from alignment in the magnetic field direction, \( \textbf{l} \). As a result, a liquid with \( |e_Z| < e_Z^s = \frac{2}{5}\tilde{u}_{so}/\lambda \omega_c^2\mathcal{J}_{N+1} \) takes one of two twofold degenerate easy-axis magnetic states, which generates a source for formation of a domain structure in a QHF. Moreover, as a feature of the SO coupling in the 2D electron system lacking inversion symmetry, for a tinier splitting, \( |e_Z| < e_Z^h = \sqrt{2}(\tilde{u}_{so}/\lambda \omega_c)^2\mathcal{J}_{N+1} \), the spin polarization acquires a helically twisted texture with a mesoscopic-scale period, \( L/\lambda = 3.25\omega_c\lambda/\tilde{u}_{so} \) in the [110] crystallographic direction. The transition of a liquid into the helical state may manifest itself in a change of the NMR line shape from the QW structure, or in the anisotropy of dissipative transport characteristics of a QHF. We also discuss topological defects in the helical state, \( \pm e/2 \)-charged dislocations, which, in pairs, constitute skyrmions.

The recipe [3,4] of describing smoothly textured QHFs’ at odd-integer filling factors is to use the 2D sigma model, which operates with the energy functional, \( \Phi(\{n(x)\}) \) of 2D electrons expressed in terms of their local excess spin polarization field \( \textbf{n}(r,t) \) (\( \{n\} = 1 \)). Locally, the QHF can be viewed as a liquid of electrons which fully occupy \((N + 1)\) LL’s with spin parallel to \( \textbf{n} \), and \( N \) LL’s with antiparallel spins. This assumes the existence of a local unitary spin transformation, \( U(\textbf{r}) \), of electron wave functions.
functions, which reduces the filling of the \((N + 1)\)st LL to a complete occupation of only states “up” with respect to the locally determined axis \(n(r)\), and which is related to the spin-polarization field as \(n' = Tr(s' \Lambda)\), where \(s'(r) = U(r) \sigma^z U^\dagger (r)\), and \(\Lambda = (1 + \sigma^z)/2\) is the electron spin-density matrix in the rotated frame. Such a liquid retains incompressibility, which is guaranteed by a large exchange energy gap \(\delta\). The derivation of a sigma model consists in the use of Hubbard-Stratonovich transformation and the saddle-point self-consistency equation, as a method to obtain an expansion of the thermodynamic potential of a liquid over small gradients of a polarization field, or, equivalently, over the matrix \(\tilde{\Omega}(r) = U(r) [-i \nabla U^\dagger (r)] \equiv [i \nabla U(r)] U^\dagger (r)\). The latter matrix appears in a local perturbation to the single-particle Hamiltonian written in the rotated spin frame,

\[
\hat{H}_\Omega = i \omega_c (a_+ \Omega_+ - \Omega_+ a_-) + (\omega_c / 2) [\tilde{\Omega}^2 - \alpha \nabla \times \tilde{\Omega}],
\]

where \(a_\pm = \pm (i \rho_x \mp \alpha \rho_y) / \sqrt{2}\) are the inter-LL operators.

\[
\Phi = \sum_{\beta} \left\{ \left( \nabla n^\beta \right)^2 / 8 - \frac{\tilde{u}_{20}^\beta + u_{20}^\beta}{2 \omega_c^2 \Lambda^2} \right\} + \frac{2 \tilde{u}_{30} \tilde{u}_{20} \tilde{u}_{30}}{\omega_c^2 \Lambda^2} n^\beta n^\gamma + \frac{\tilde{u}_{30} \tilde{u}_{20}}{\omega_c^2 \Lambda^2} \cdot \nabla n^\beta + E_{sk},
\]

where \(\hat{H}_\Omega\) is the planar component of the spin polarization field, and \(\tilde{n}^\beta = n^\beta / n, \tilde{n}^\gamma = n^\gamma / n\). To obtain \(\Phi\) in Eq. (1), we have extended perturbative expansion up to the terms \(3 \nabla^2, 3 (\tilde{u}_{30} / \omega_c \Lambda) \nabla, \) and \(3 (\tilde{u}_{30} / \omega_c \Lambda)^2\). The exchange factor,

\[
\tilde{N}_{\nu+1} = \int_0^\infty dz V(\sqrt{2}z) e^{-z} \left\{ \sum_{M = -N, N+1} M [L_M(z) L_M(\bar{z}) - L_{M-1}(z) L_{M-1}(\bar{z})] \right\} = \frac{e^2 \sqrt{2\pi}}{x^2} \theta_{N+1},
\]

was calculated for each odd-integer filling \(\nu = 2N + 1\) \((N = 0, 1, 2, \ldots)\); \(L_M^\beta(z)\) are the generalized Laguerre polynomials. For \(V(r) = e^2 / r \chi\), which is a reasonable approximation for the 2D electron interaction in a narrow QW \((\nu \ll \lambda\) ), \(\theta_1 = 1/2, \theta_2 = 7/16, \theta_3 = 0.57, \theta_4 = 0.67\), and \(\theta_5 = 0.76\). Since the relevant part of the SO coupling is off-diagonal with respect to the Landau level number, it appears only in the second order of a perturbation theory, or due to its mixing with the \(\hat{H}_\Omega\) term [21]. The single-particle spin-splitting, \(\varepsilon_Z\) in Eq. (1) is corrected by the effect of the 2D single-particle SO coupling: \(\varepsilon_Z = \mu g B - \alpha n (\tilde{u}_{30} - u_{30}^\beta) / \omega_c \Lambda^2\). We also include into \(\Phi(n, \nabla n)\) the topological term and the Coulomb energy of additional charges, \(\rho(x)\), in order to discuss charged skyrmion-type textures,

\[
E_{sk} = \sum_{\nu=1}^{N+1} \frac{1}{2} \rho + \int dx' \rho(x) \frac{V(x - x')}{2} \rho(x');
\]

\[
\rho(x) = \frac{-\alpha}{8\pi} e^{\beta \gamma \delta} e^{ij} n^\beta \partial_i n^\gamma \partial_j n^\delta.
\]

From a phenomenological point of view, thermodynamic potential \(\Phi(n, \nabla n)\) in Eq. (1) describes an easy-axis ferromagnet with square 2D Bravis lattice and broken inversion symmetry, and in a perpendicular magnetic field. It contains all terms in the magnetization energy allowed by the crystalline symmetry of the [001]-grown quantum well in a zinc-blend-type semiconductor [22]. The sketched above microscopic derivation of \(\Phi(n, \nabla n)\) was necessary to obtain the values of coefficients in front of the phenomenologically allowed invariants composed of \(\rho\) and \(\nabla\). The first term in \(\Phi\) describes spin stiffness. The second term determines an easy-axis anisotropy along \(l_z = [110] / \sqrt{2}\). For structures with \(|u_{30}| \gg u_{20}\), it rather defines a weakly anisotropic easy plane for spin polarization, which competes with the Zeeman energy term. Such a competition resumes itself in the deviation of polarization from a fully \(l_z\)-aligned state at \(|\varepsilon_Z| < e^z\).

\[
e^z = 2 (u_+ / \omega_c \Lambda)^2 \theta_{N+1}, \quad u_\pm = \tilde{u}_{30} \pm u_{30},
\]

As a function of a varying \(\varepsilon_Z\), this can be viewed as a second order phase transition into the easy-axis state,

\[
n_\pm = -(\varepsilon_Z / e^z) l_z \pm 1 \sqrt{1 - (\varepsilon_Z / e^z)^2},
\]

across which the symmetry between \pm magnetic directions gets spontaneously broken [18]. Since for \(|\varepsilon_Z| < e^z\) both easy-axis configurations, \(n_+\) and \(n_-\), have the same energy density,

\[
\Phi_{\pm} = \frac{3}{4 \pi \lambda^2} \left( u_+ / \omega_c \Lambda \right)^2 \left[ \begin{array}{c} 1 + (\varepsilon_Z / e^z)^2 \end{array} \right].
\]
the easy-axis state of a QHF tends to acquire a domain structure: by splitting dynamically into the set of mesoscopic-size regions with opposite polarization projections onto the [110] axis. The latter possibility has to affect the skyrmion-dominated dissipative transport properties of a liquid. The matter is that the activation energy of a skyrmion-antiskyrmion pair confined to the domain wall is lower than in the 2D bulk [21]. In fact, the larger the difference in the polarization between two domains, the more skyrmion is energetically confined to it. Since \( n_+ \)-polarized states are degenerate, the areas covered by \( n_+ \) and \( n_- \) domains are statistically equal, so that the network of better conducting domain walls (with a lower activation energy for thermally excited carriers) forms a percolation cluster, thus resulting in a continuous decline in the activation energy for the macroscopic \( \sigma_{xx} \) which would follow the decrease of \( e_Z \).

A further analysis of the functional in Eq. (1) extended onto the limit of \( e_Z = 0 \) shows that, apart from the domain structure formation, there is another reason for the field \( n(r) \) to be inhomogeneous. The fourth term [22] in Eq. (1) tends to twist the polarization field of a QHF into helical texture,

\[
n(r) = l \sin[\phi(r)] + l_z \cos[\phi(r)]. \tag{6}
\]

The latter is characterized by the helicity plane built upon two unit vectors, \( l = (l^x, l^y, 0) \) and \( l_z \), spatial orientation \( m \), and period \( L \), \( \phi (r + m L) = \phi (r) + 2\pi \). As a variational approximation, one can use \( n(r) \) in Eq. (6) with \( \phi (r) = m r / L \), treating \( l, m \), and \( L \) as minimization parameters. The result can be viewed as an image of a spoke in a wheel rolling in the direction of \( m \), with \( \phi \) being the integral angle encircled by a spoke, so that we shall call \( \phi \) the “helicity phase.” The energy density of an optimal variational state,

\[
m = 1 = 1 - \lfloor \text{[110]} / \sqrt{2}, L = \pi \lambda^2 / u_+, \tag{7}\n\]

\[
\Phi_h = -\frac{\Gamma_{N+1}}{4\pi \lambda^2} \left( \frac{u_+ / \omega c}{\omega c} \right)^2 + \frac{1}{2} \left( \frac{u_- / \omega c}{\omega c} \right)^2 \right], \tag{8}
\]

is lower than the energy \( \Phi_e(e_Z = 0) \) of a homogeneous easy-axis configuration in Eq. (5). The optimal variational state provides us with values of \( \Phi_h \) and \( L \) which are very close to the \( n \)-field distribution that really minimizes [23] the functional \( F \) in Eq. (1). For the experimentally relevant example of \( u_+ = u_- \) (i.e., \( \tilde{u}_{so} \gg u_{so} \)), \( \Phi_h = \frac{1}{2} \tilde{u}_{so}(\lambda / \omega c)^2(\Gamma_{N+1} / 4\pi \lambda^2) \) and \( L = 3.25 \omega c / \tilde{u}_{so} \). We, therefore, assess the stability of a helical state, against the easy-axis one, on the basis of the energetics of variational helical texture with parameters given by Eq. (7).

Because of the difference in symmetry of helical and easy-axis states, which cannot be continuously transformed one into another, the transformation between them can be viewed as a first-order phase transition. The condition for such a transition, \( \Phi_h = \Phi_e(e_Z) \), determines critical value of Zeeman splitting, \( e_Z^c \) estimated as

\[
e_Z^c = \sqrt{2}(u_+ u_- / \omega c \lambda B_{\text{H}} \lambda_{N+1}) = e_Z^c / \sqrt{2}. \tag{9}\n\]

The helical state formation can manifest itself in several observations. For example, the local value of the Knight shift [6], \( \delta_{hf} \) in the spin-splitting of Ga and As nuclei located in the QW acquires an alternate coordinate-dependent sign, modifying the NMR lineshape, \( I(\delta) \). Locally, the NMR shift \( \delta = (\omega - \omega_0) / \delta_{hf} \) is due to the hyperfine interaction of nuclear spins with fully polarized electrons, with \( \delta_{hf} \) being its maximal value just in the QW center. In a homogeneously polarized gas, the NMR line from the QW has a double-peak structure [6], \( I(1 > \delta > 0) \propto (|\delta_1 - \delta_2|)^{-1/2} \), with a distinct satellite at \( \delta = 1 \) split by the hyperfine coupling. In the easy-axis configuration, Eq. (4), the double-peak structure persists, with a reduced maximal splitting: \( \delta_{hf} \rightarrow (e_Z / e_Z^c) \delta_{hf} \), as far as \( n^2 = -e_Z / e_Z^c \). On the contrary, in a helical phase, this has to transform into a single broadly tailed resonance with a non-Lorenzian shape approximated by \( I(\delta) \propto |\delta|^{-1/2} \) for \( 1 > \delta > -1 \).

The anisotropy of transport characteristics of a QHF with respect to [110] and [110] crystallographic directions may be another feature of the helical state. Speaking about dissipative conductivity formed by thermally activated electron-hole pairs at the spin-split LL’s, this can be understood after taking into account that charge-carrying excitations determined in a locally rotated spin frame [adjusted to \( n(r) \)] are subjected to a smooth (\( L \gg \lambda \)) potential due to Zeeman energy, \( (e_Z / 2)\cos(|x - y| / \sqrt{2}L) \). Therefore, at low temperatures, \( T < e_Z \) the dissipative conductivity \( \sigma_{xx} \) along \( m || [110] \) would be suppressed, as compared to \( \sigma_{xx} \) (across \( m \)).

For the dissipative transport dominated by skyrmions [5], the difference between \( \sigma_{xx} \) and \( \sigma_{xx} \) is to be the result of the anisotropy of a skyrmion itself. In fact, the form of a skyrmion in a helically twisted texture is quite complex: In a periodic system, these are dislocations which represent the very elementary topological defects, rather than skyrmions. The periodicity of a helical texture in Eq. (6) is controlled by the helicity phase \( \phi_0(mr / \sqrt{2}) \) in Eqs. (6) and (7). One missing (or extra) period in one half of a plane, as compared to the other half (a dislocation) is equivalent to the phase shift of \( \pm 2\pi \) accumulated at large distances from the dislocation core, thus resulting in the winding number \( D = \pm 1 \). On the other hand, we assume that the dislocation core is not singular. To illustrate the topology of a nonsingular core, let us draw a large-radius circle around a dislocation. At large distances, where helical structure is not perturbed, such a contour maps into the equator of a unit sphere and encircles it \( N \) or \((N - 1)\) times, depending on which one of two semicircles is traced: drawn above, or below the dislocation.
Upon moving the upper half of a contour down through the dislocation, an extra loop encircling the unit sphere equator should continuously disappear. The latter is possible if the contour image slips through either the $+1$, or, alternatively, $-1$ pole of a unit sphere, which can be modeled by such a field configuration $\mathbf{n}(\mathbf{r}) = 1_nz + 1 بالن + 1 بالن$.

$$n^z + in^- = e^{i\phi + iD\varphi}\sqrt{1 - (n^z)^2}, \left\{ n^+(0) = \pm 1, n^+(r \to \infty) = 0, \right.$$ 

where $(r, \varphi)$ are polar coordinates calculated from the dislocation center, $r = 0$. The image of a 2D plane provided by $\mathbf{n}(\mathbf{r})$ maps onto only one half of a sphere $|\mathbf{n}| = 1$, so that the dislocation core is characterized by an additional topological number, $\theta = n^+(0) = \pm 1$ distinguishing between “left” and “right” semispheres. Using $\rho(x)$ in Eq. (2), we find that the core of a dislocation ($D = 1$) or antidislocation ($D = -1$) carry a half-integer electric charge $\int d\mathbf{x} \rho(x) = \frac{\pi}{2}\vartheta D$. However, an isolated dislocation has a logarithmically large energy.

$$\mathcal{F}(D = \pm 1) = \int \frac{dx}{2\pi} \frac{3N+1}{8} (\nabla \varphi)^2 \approx \frac{3N+1}{8} \ln(rk_0),$$

and, at low temperatures, dislocations and antidislocations have to form pairs bound by a long-range (logarithmic) attraction, except, maybe, for a possible Kosterlitz-Thouless melting effect. Since both dislocation and antidislocation in a bound pair may be equally charged, such a pair, $(D, \vartheta)$ and $(-D, -\vartheta)$ together constitute a skyrmion.

The result of the above analysis of phases of a ferromagnetic quantum Hall effect liquid in a narrow QW can be summarized as follows. Upon decreasing the single-particle Zeeman splitting, $|eZ|$ (e.g., by pressure), spin polarization of a liquid starts to acquire at $|eZ| = eZ^h$ the easy-axis configuration, which is followed by the abrupt fall into a helical state at $|eZ| = eZ^h = eZ^h/\sqrt{2}$. Using the bulk SO coupling parameter $\gamma = 25\text{eV}\text{Å}^3$, as a reference, we estimate for the $\nu = 1$ liquid in a 68-Å-wide GaAs/AlGaAs quantum well structure with a carrier density $2.8 \times 10^{11} \text{cm}^{-2}$ studied by Maude et al. [8] that $eZ^h/(e^2/\chi \lambda) = 1.5 \times 10^{-3}$, and $eZ^h/(e^2/\chi \lambda) \sim 1 \times 10^{-3}$, which roughly fits into the range of a variable Zeeman energy in Ref. [8], where the dissipative transport activation has been drastically affected by pressure. The helical texture period estimated for the same parameters is $\mathcal{L} \sim 5 \times 10^3 \text{Å}$. Note that, according to Eqs. (9) and (3), the parametric range of pressures where helical and easy-axis phases are stable is broader for higher odd-integer filling factors in the same density structure.

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[18] Our attention to the relevance of the easy-axis anisotropy has been turned by Yu. Nazarov and by N. Cooper.
[22] This expression for the energy density is given neglecting $(\mathcal{N}+1)/2\mathbf{\omega}_\lambda^2\delta^{n}(\mathbf{u}_\alpha - n^\beta\mathbf{n}) + \mathbf{u}_\alpha(\mathbf{u}_\alpha - n^\beta\mathbf{n})$, which can be transformed into the edges term.
[23] To minimize the energy, we first find such a helicity phase, $\phi_0(\mathbf{r} + \mathbf{m} \mathcal{L}) = \phi_0(\mathbf{r}) + 2\pi$ that optimizes $\Phi$ for a given period $\mathcal{L}$. Then, we minimize $\Phi(\mathcal{L})$ with respect to $\mathcal{L}$. The optimal $\phi_0$ for $eZ = 0$ is determined by the equation $-1/(4\pi^2)\delta^2 \phi_0 - (u_\perp/\lambda \mathbf{\omega}_\lambda)\sin\phi_0 \cos\phi_0 = 0$, with first integral $\mathcal{A} = (\lambda \mathbf{\omega}_\lambda/2u_\perp)^2(\delta^2 \phi_0)^2 \cos^2 \phi_0$, $\eta = \mathbf{r} \cdot \mathbf{m}$, related to the period as $2\mathcal{L} u_\perp/\lambda \mathbf{\omega}_\lambda = \int_0^{\mathcal{L}} d\phi \sqrt{\mathcal{A} + \cos^2 \phi}$. Using these relations, we find the energy dependence $\Phi_0(\mathcal{L})$ and formally minimize it, $d\Phi_0(\mathcal{L})/d\mathcal{L} = 0$, generating one more integral relation, $\int_0^{\mathcal{L}} d\phi \sqrt{\mathcal{A} + \cos^2 \phi} = 2\pi u_+ / u_-$. For $u_+ \approx u_- \approx \tilde{u}$, we find $\mathcal{A} \approx 0.532$, $\mathcal{L} \approx 3.25u_+ / \tilde{u}$, and $\Phi_0 \approx -1.53(\tilde{u}/\lambda \mathbf{\omega}_\lambda)^2(3\mathcal{N}+1)/(4\pi \lambda^2)$. 

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