Shot-Noise in Non-Degenerate Semiconductors with Energy-Dependent Elastic Scattering

H. Schomerus[1], E.G. Mishchenko[1,2], and C.W.J. Beenakker[1]

[1] Instituut-Lorentz, Universiteit Leiden, P. O. Box 9506, 2300 RA Leiden, The Netherlands
[2] L. D. Landau Institute for Theoretical Physics, Kosygin 2, Moscow 117334, Russia

Abstract. We investigate current fluctuations in non-degenerate semiconductors, on length scales intermediate between the elastic and inelastic mean free paths. The shot-noise power P is suppressed below the Poisson value $P_{\mathrm{Poisson}} = 2e\bar{I}$ (at mean current $\bar{I}$) by the Coulomb repulsion of the carriers. We consider a power-law dependence of the elastic scattering time $\tau \propto \varepsilon^{\alpha}$ on kinetic energy ε and present an exact solution of the non-linear kinetic equations in the regime of space-charge limited conduction. The ratio P/P_{Poisson} decreases from 0.38 to 0 in the range $-\frac{1}{2} < \alpha < 1$.

1 Introduction

The noise power P of current fluctuations in an electron gas in thermal equilibrium (at temperature T) is related by the Johnson-Nyquist formula $P = 4kTG$ (with k Boltzmann's constant) to the linear-response conductance $G = \lim_{V\to 0} d\bar{I}/dV$ (with $\bar{I}$ the mean current in response to an applied voltage V). This formula can be generalized to a large applied voltage, $P = 4kT(V/\bar{I})(d\bar{I}/dV)^2$, provided the electron gas remains in local equilibrium with the lattice. Local equilibrium requires inelastic scattering. When the conductor is shorter than the inelastic mean free path l_{in} and the potential drop V is large enough, the Johnson-Nyquist formula no longer applies and a measurement of current noise (then also called shot noise) reveals more detailed information about the transport of charge carriers—in particular about their correlations. The maximal noise level $P_{\mathrm{Poisson}} = 2e\bar{I}$ is attained in absence of all correlations (both in the injection process as well as in the subsequent transport). Examples are vacuum diodes at large bias in absence of space-charge effects and tunneling diodes with low transmissivity.

Here we consider the transport through a disordered semiconductor of length L terminated by two metal contacts, under the conditions of elastic scattering ($l \ll L \ll l_{\mathrm{in}}$, with l the elastic mean free path). In a degenerate conductor correlations are induced by the Pauli exclusion principle (for a review of the theory of shot noise in this situation see Ref. [1]) and the shot noise has the universal value $P = \frac{1}{3}P_{\mathrm{Poisson}}$ [2], [3].

At low carrier concentration the electron gas is non-degenerate, and the Pauli principle is ineffective. Because carriers can now accumulate, giving

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rise to space-charge effects, they become correlated through Coulomb repulsion. This is the situation which we want to study presently. In a recent Monte-Carlo simulation [4] a shot-noise suppression factor of about $P/P_{\text{Poisson}} = 1/3$ was found in the regime of space-charge limited transport; an energy-independent elastic scattering rate was assumed. The coincidence with the noise level obtained in the degenerate situation attracted a lot of attention [5]. The degree of universality is less pronounced here since the number actually depends on the geometry and dimensionality—as well as the scattering mechanism [6], [7], [8].

In Ref. [6] the problem was investigated for an energy-independent elastic scattering time τ, using the kinetic theory of non-equilibrium fluctuations (reviewed in Ref. [9]). The non-linear kinetic equations were solved in a certain approximation (the drift approximation), with the result $P/P_{\text{Poisson}} = 0.3410$. In Ref. [7] we obtained an exact solution, giving $P/P_{\text{Poisson}} = 0.3097$, and also considered a power-law dependence $\tau \sim \varepsilon^{\alpha}$ on the kinetic energy ε. For $\alpha = -\frac{1}{2}$ (corresponding to short-range impurity scattering or quasi-elastic acoustic phonon scattering [10]) we found the exact result $P/P_{\text{Poisson}} = 0.3777$. For other values of α we only presented results within the drift approximation. In this work we derive the exact solution in the range $-\frac{1}{2} < \alpha < 1$. As we will discuss, α should be in this range for space-charge limited conduction to be realized.

2 The Drift-Diffusion Equation

We consider a three-dimensional conductor of length L and cross-sectional area A terminated by two contacts. The equilibrium density ρ_{eq} of charge carriers (charge e, effective mass m) in the decoupled conductor is assumed to be much lower than the density ρ_c of those carriers that are energetically allowed (at a given voltage V) to enter the conductor from the contacts. (A possible realization would be an intrinsic or barely doped semiconductor between two metal contacts or two heavily doped semiconducting regions.) The dielectric constant of the conductor is κ. The temperature T is assumed to be so high that the electron gas is degenerate, and a large voltage drop $V \gg kT/e$ is maintained between the contacts. Transport is assumed to be diffusive and elastic, $l < L < l_{\text{in}}$. We assume a power-law energy dependence

$$\tau(\varepsilon) = \tau_0 \varepsilon^{\alpha} \tag{1}$$

of the elastic scattering time on the kinetic energy ε. We want to calculate the zero-frequency component

$$P = 2 \int_{-\infty}^{\infty} \mathrm{d}t' \, \overline{\delta I(t) \delta I(t+t')} \tag{2}$$

of the noise power of the fluctuations $\delta I(t)$ of the electric current $I(t) = \bar{I} + \delta I(t)$ around its mean $\bar{I}$.

We use Cartesian coordinates x, y, z with x parallel to the conductor (the current source is at $x = 0$, the drain at $x = L$). To linear order in the fluctuations, the transverse coordinates can be ignored. In the zero-frequency limit the current is independent on x because of the continuity equation and is given by the drift-diffusion equation [6], [7]

$$I(t) = -\frac{\partial}{\partial x}\int d\varepsilon\, D(\varepsilon)\rho(x,\varepsilon,t) + E(x,t)\int d\varepsilon\, \mathcal{F}(x,\varepsilon,t)\frac{d\sigma(\varepsilon)}{d\varepsilon} + \delta J(x,t). \quad (3)$$

The electric field $E(x,t)$ is related to the laterally integrated charge density $\rho(x,t)$ by the Poisson equation

$$\kappa\frac{\partial}{\partial x}E(x,t) = \frac{1}{A}\rho(x,t), \quad (4)$$

where we omitted the low background charge density $-\rho_{\mathrm{eq}}$. The fluctuating source $\delta J(x,\varepsilon,t)$ accounts for the stochasticity of individual scattering events and has the correlator

$$\overline{\delta J(x,t)\delta J(x',t')} = 2A\delta(t-t')\delta(x-x')\int \mathrm{d}\varepsilon\, \sigma(\varepsilon)\bar{\mathcal{F}}(x,\varepsilon). \quad (5)$$

Here and in Eq. (3), $\mathcal{F}(x,\varepsilon,t) = \rho(x,\varepsilon,t)/e\nu(\varepsilon)$ with the density of states $\nu(\varepsilon) = 4\pi m(2m\varepsilon)^{1/2} = \nu_0\varepsilon^{1/2}$ (we set Planck's constant $h \equiv 1$). The conductivity $\sigma(\varepsilon) = e^2\nu(\varepsilon)D(\varepsilon) = \sigma_0\varepsilon^{\alpha+3/2}$ is the product of the density of states and the diffusion constant $D(\varepsilon) = v^2\tau/3 = D_0\varepsilon^{\alpha+1}$.

3 Space-Charge Limited Conduction

For a large voltage drop V between the two metal contacts and a high carrier density ρ_c in the contacts, the charge injected into the semiconductor is much higher than the equilibrium charge ρ_{eq}, which can then be neglected. For sufficiently high V and ρ_c the system enters the regime of space-charge limited conduction [11], defined by the boundary condition

$$E(x,t) = 0 \quad \text{at} \quad x = 0. \quad (6)$$

Eq. (6) states that the space charge $Q = \int_0^L \rho(x)\,\mathrm{d}x$ in the semiconductor is precisely balanced by the surface charge at the current drain. At the drain we have the absorbing boundary condition

$$\rho(x,t) = 0 \quad \text{at} \quad x = L. \quad (7)$$

With this boundary condition we again neglect ρ_{eq}.

To determine the electric field inside the semiconductor we proceed as follows. Since scattering is elastic, the total energy $u = \varepsilon - e\phi(x,t)$ of each carrier is preserved. The potential gain $-e\phi(x,t)$ (with $E = -\partial\phi/\partial x$) dominates over the initial thermal excitation energy of order kT almost throughout the whole semiconductor; only close to the current source (in a thin boundary layer) this is not the case. We can therefore approximate the kinetic energy $\varepsilon \approx -e\phi$ and introduce this into $D(\varepsilon)$ and $d\sigma/d\varepsilon$. Substituting into Eq. (3) one obtains

$$\mathcal{F}(x,t) \approx e \int_x^L dx' \, \frac{I(t) - \delta J(x,t)}{\sigma_0 [-e\phi(x',t)]^{\alpha+3/2}}, \tag{8}$$

$$\rho(x,t) \approx \frac{[-e\phi(x,t)]^{1/2}}{D_0} \int_x^L dx' \, \frac{I(t) - \delta J(x,t)}{[-e\phi(x',t)]^{\alpha+3/2}}, \tag{9}$$

where the absorbing boundary conditions have been used. From the Poisson equation (4) we find the third-order, non-linear, inhomogeneous differential equation

$$2(-\phi)^{\alpha}\phi'\phi'' + 4(-\phi)^{\alpha+1}\phi''' = B\bar{I}[1 + \delta i(x,t)], \tag{10}$$

$$\delta i(x,t) = \frac{I(t) - \delta J(x,t)}{\bar{I}}, \tag{11}$$

for the potential profile $\phi(x,t)$. Primes denote differentiation with respect to x, and $B = 6/e^{\alpha}\mu_0\kappa A$ with $\mu_0 = e\tau_0/m$.

Since the potential difference V between source and drain does not fluctuate, we have the two boundary conditions $\phi(0,t) = 0$, $\phi(L,t) = -V$. Eqs. (6) and (7) imply two additional boundary conditions, $\phi'(0,t) = 0$, $\phi''(L,t) = 0$.

The differential equation (10) and the accompanying boundary conditions possess two remarkable scaling properties: The product $B\bar{I}$ of material parameters and mean current $\bar{I}$ and the length L can be eliminated by introduction of the scaled potential

$$\chi(x,t) = -\left(L^3 B\bar{I}\right)^{-1/(\alpha+2)} \phi(xL,t). \tag{12}$$

The rescaled differential equation reads

$$2\chi^{\alpha}\chi'\chi'' - 4\chi^{\alpha+1}\chi''' = 1 + \delta i, \tag{13}$$

which has to be solved with the boundary conditions $\chi(0,t) = 0$, $\chi(1,t) = \left(L^3 B\bar{I}\right)^{-1/(\alpha+2)} V$, $\chi'(0,t) = 0$, $\chi''(1,t) = 0$. The scaling properties entail that the shot-noise suppression factor depends only on the exponent α, but no longer on the parameters L, A, V, τ_0, and κ.

We will solve this boundary value problem for $\chi = \bar{\chi} + \delta\chi$, first for the mean (Section 4) and then for the fluctuations (Section 5), in both cases neglecting terms quadratic in $\delta\chi$.

4 Average Profiles and the Current-Voltage Characteristic

The averaged equation (13) for the rescaled mean potential $\bar{\chi}(x)$ reads

$$2\bar{\chi}^{\alpha}\bar{\chi}'\bar{\chi}'' - 4\bar{\chi}^{\alpha+1}\bar{\chi}''' = 1. \tag{14}$$

We seek a solution which fulfills the three boundary conditions $\bar{\chi}(0) = 0$, $\bar{\chi}'(0) = 0$, $\bar{\chi}''(1) = 0$. The value of $\bar{\chi}$ at the current drain determines the current-voltage characteristic

$$\bar{I}(V) = \frac{1}{L^3 B}\left(\frac{V}{\bar{\chi}(1)}\right)^{\alpha+2}. \tag{15}$$

We now construct $\bar{\chi}(x)$. The function $\bar{\chi}_0(x) = a_0 x^{\beta}$ with $\beta = 3/(2+\alpha)$ and $a_0 = [2\beta(\beta-1)(4-\beta)]^{-\beta/3}$ solves the differential equation and satisfies the boundary conditions at $x = 0$, but $\bar{\chi}_0''(x) \neq 0$ for any finite x. We substitute into Eq. (14) the ansatz $\bar{\chi}(x) = \sum_{l=0}^{\infty} a_l x^{\gamma l+\beta}$, consisting of $\bar{\chi}_0(x)$ times a power series in x^{γ}, with γ a positive power to be determined. This ansatz proves fruitful since both terms on the left-hand side of Eq. (14) give the same powers of x, starting with order x^0 in coincidence with the right-hand side. By power matching one obtains in first order the value for a_0 given above. The second order leaves a_1 as a free coefficient, but fixes the power $\gamma = (8 - 5\beta + \sqrt{-32 + 40\beta + \beta^2})/4$. The coefficients a_l for $l \geq 2$ are then given recursively as a function of a_1, which is finally determined from the condition $\bar{\chi}''(1) = 0$.

In Fig. 1 the profiles of the potential $\bar{\phi} \propto \bar{\chi}$, the electric field $\bar{E} \propto \bar{\chi}'$, and the charge density $\bar{\rho} \propto \bar{\chi}''$ are plotted for various values of α. The coefficient $\bar{\chi}(1)$ appearing in the current-voltage characteristic (15) can be read off from this plot. The behavior at the current source changes qualitatively at $\alpha = -\frac{1}{2}$ (see Section 7).

5 Fluctuations

The rescaled fluctuations $\delta\chi(x,t) = \psi(x,t)$ fulfill the linear differential equation

$$\begin{aligned}\mathcal{L}[\psi] = {} & -4\bar{\chi}^{\alpha+1}\psi''' + 2\bar{\chi}^{\alpha}\bar{\chi}'\psi'' + 2\bar{\chi}^{\alpha}\bar{\chi}''\psi' \\ & + \left[2\alpha\bar{\chi}^{\alpha-1}\bar{\chi}'\bar{\chi}'' - 4(\alpha+1)\bar{\chi}^{\alpha}\bar{\chi}'''\right]\psi = \delta i(t).\end{aligned} \tag{16}$$

The solution of the inhomogeneous equation is found with help of the three independent solutions of the homogeneous equation $\mathcal{L}[\psi] = 0$, $\psi_1(x) = \bar{\chi}'(x)$, $\psi_2(x) = \bar{\chi}(x) - (x/\beta)\bar{\chi}'(x)$, and

$$\psi_3(x) = \psi_1(x)\int_x^1 \mathrm{d}x' \,\frac{\bar{\chi}^{1/2}(x')\psi_2(x')}{\mathcal{W}^2(x')} - \psi_2(x)\int_x^1 \mathrm{d}x' \,\frac{\bar{\chi}^{1/2}(x')\psi_1(x')}{\mathcal{W}^2(x')}, \tag{17}$$

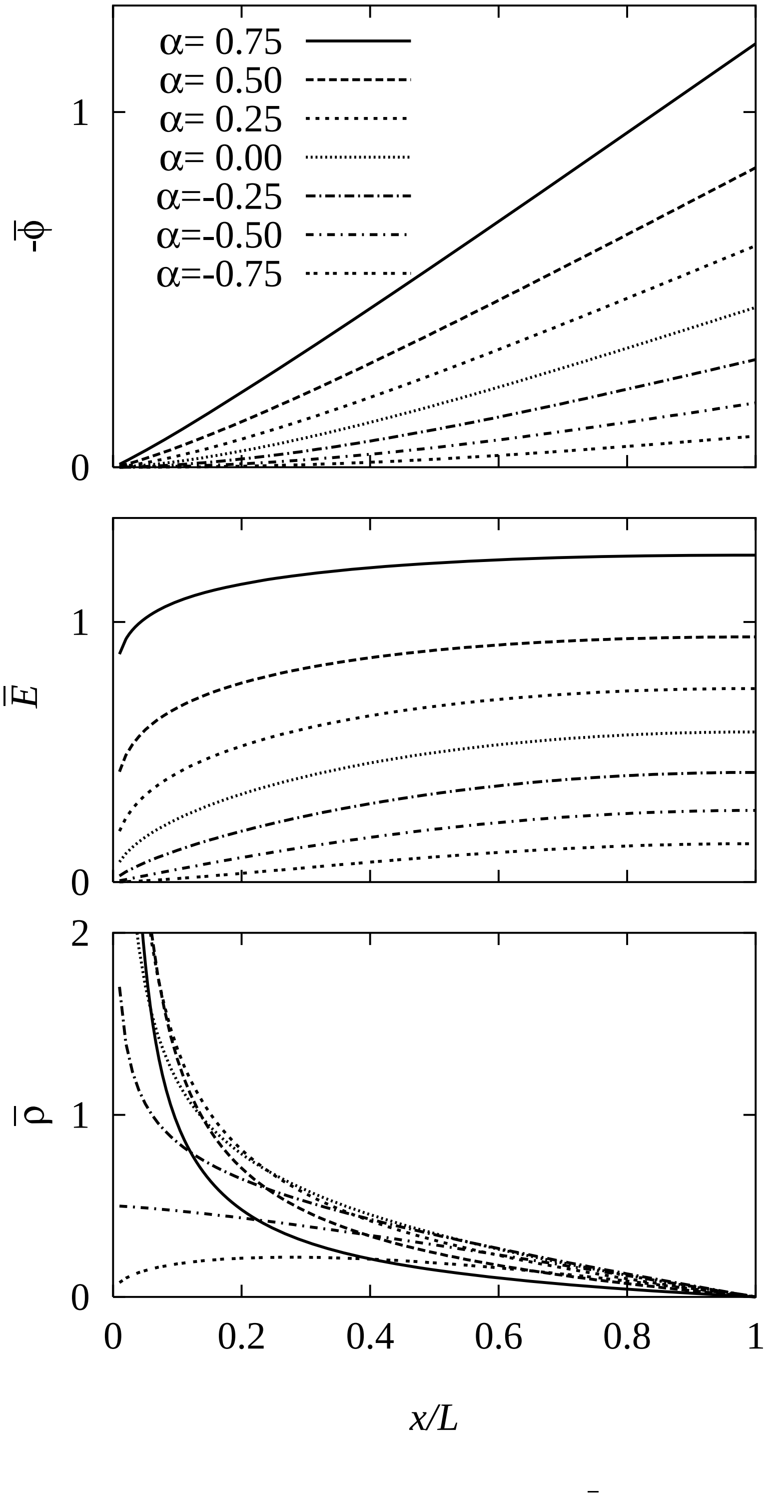

Fig. 1. Profile of the mean electrical potential $\bar{\phi}$ [in units of $(L^3B\bar{I})^{1/(\alpha+2)}$, with $B = 6m/e^{\alpha+1}\tau_0\kappa A$], the electric field $\bar{E}$ [in units of $(L^3B\bar{I})^{1/(\alpha+2)}/L$], and the charge density $\bar{\rho}$ [in units of $\kappa(L^3B\bar{I})^{1/(\alpha+2)}/L^2$], following from Eq. (14) for different values of α.

where we have defined $\mathcal{W}(x) = \psi_1(x)\psi_2'(x) - \psi_1'(x)\psi_2(x)$. The special solution which fulfills $\psi(0,t) = \psi'(0,t) = \psi(1,t) = 0$ is

$$\psi(x,t) = \int_0^1 \mathrm{d}x' \, \frac{\bar{\chi}^{1/2}(x')}{\mathcal{W}^2(x')} \Big[\Theta(x-x')\psi_1(x)\psi_2(x') + \Theta(x'-x)\psi_1(x')\psi_2(x) - \frac{\psi_1(1)}{\psi_2(1)}\psi_2(x)\psi_2(x') \Big] \int_0^{x'} \mathrm{d}x'' \, \frac{\delta I(t) - \delta J(x'',t)}{4\bar{I}} \, \frac{\mathcal{W}(x'')}{\bar{\chi}^{\alpha+3/2}(x'')} . \tag{18}$$

The condition $\psi''(1,t) = 0$ relates the fluctuating current δI to the Langevin current δJ. The resulting expression is of the form

$$\delta I(t) = \mathcal{C}^{-1} \int_0^L \mathrm{d}x \, \delta J(x,t) \mathcal{G}(x), \tag{19}$$

with the definitions $\mathcal{C} = \int_0^1 \mathrm{d}x \, \mathcal{G}(x)$,

$$\mathcal{G}(x) = \frac{\mathcal{W}(x)}{\bar{\chi}^{\alpha+3/2}(x)} \left(1 + \frac{(1-1/\beta)\bar{\chi}'^2(1)}{4\bar{\chi}^{\alpha+1/2}(1)\psi_2(1)} \int_x^1 \mathrm{d}x' \, \frac{\bar{\chi}^{1/2}(x')\psi_2(x')}{\mathcal{W}^2(x')} \right) . \tag{20}$$

The shot-noise power is found by substituting Eq. (19) into Eq. (2) and invoking the correlator (5) for the Langevin current. This results in

$$P = 2 \int_0^L \mathrm{d}x \, \left(\frac{\mathcal{G}(x)}{\mathcal{C}} \right)^2 \mathcal{H}(x) \tag{21}$$

with $\mathcal{H}(x) = 2A \int \mathrm{d}\varepsilon \, \sigma(\varepsilon) \bar{\mathcal{F}}(x,\varepsilon) \approx 2\sigma_0[-e\bar{\phi}(x)]^{\alpha+3/2} \bar{\mathcal{F}}(x)$. Eq. (8) gives

$$\mathcal{H}(x) = 2e\bar{I}\bar{\chi}^{\alpha+3/2}(x) \int_x^1 dx' \, \frac{1}{\bar{\chi}^{\alpha+3/2}(x')} = 4P_{\mathrm{Poisson}} \bar{\chi}^{\alpha+1}(x) \bar{\chi}''(x), \tag{22}$$

where we integrated with help of Eq. (14) and used $\bar{\chi}''(1) = 0$.

In Fig. 2 we plot the ratio P/P_{Poisson} as a function of the parameter α (solid curve). The shot-noise suppression factor $P/P_{\mathrm{Poisson}} = 0.3777$ for $\alpha = -\frac{1}{2}$ and goes to zero as $\alpha \to 1$.

6 Drift Approximation

A simple formula for the shot-noise suppression factor can be found when one neglects the diffusion term in Eq. (3) and considers instead of Eq. (13) the corresponding differential equation $(4\alpha + 6)\chi^\alpha \chi' \chi'' = 1 + \delta i$. The is the drift approximation of Ref. [6]. The order of the differential equation is reduced by one, so that we also have to drop one of the boundary conditions. The absorbing boundary condition $\chi''(1,t) = 0$ is the most reasonable candidate, because even for the resulting mean profile $\bar{\chi}(x) = b_0 x^\beta$ with $\beta = 3(2+\alpha)^{-1}$ and $b_0 = [\beta^2(\beta-1)]^{-\beta/3}$ most carriers remain concentrated close to the current source. The differential equation for the fluctuations $\alpha\psi/\bar{\chi} + \psi'/\bar{\chi}' +$

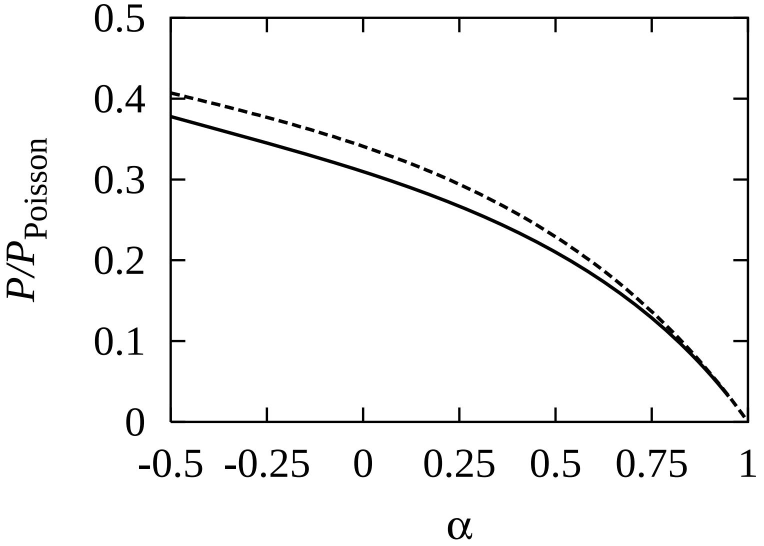

Fig. 2. Shot-noise power P as a function of α. The exact result (solid curve) is compared with the approximate result (24) (dashed curve).

$\psi''/\bar{\chi}'' = \delta i$ can be solved with help of the homogeneous solutions $\psi_1(x) = x^{\beta-1}$ and $\psi_2(x) = x^{3-2\beta}$. The inhomogeneous solution that fulfills $\psi(0,t) = 0$, $\psi'(0,t) = 0$ is

$$\psi(x,t) = b_0 \frac{\beta(\beta-1)}{4-3\beta} \int_0^x dx' \left[x^{3-2\beta} x'^{3\beta-4} - x^{\beta-1} \right] \delta i(x',t). \tag{23}$$

We demand that the voltage does not fluctuate, $\psi(1,t) = 0$, and obtain Eq. (19) with now $\mathcal{G}(x) = 1 - x^{3\beta-4}$. The shot noise power is finally found from Eq. (21) with $\mathcal{H}(x) = P_{\text{Poisson}} x^{3-\beta/2} \int_x^1 dx' \, x'^{\beta/2-3}$,

$$P/P_{\text{Poisson}} = \frac{6(\alpha-1)(\alpha+2)(16\alpha^2+36\alpha-157)}{5(2\alpha-5)(8\alpha-17)(13+8\alpha)}. \tag{24}$$

This is the dashed curve in Fig. 2.

7 Discussion

The shot-noise suppression factor P/P_{Poisson} varies from 0.38 to 0 in the range $-\frac{1}{2} < \alpha < 1$, which includes the case of an energy-independent elastic scattering rate ($\alpha = 0$, $P/P_{\text{Poisson}} = 0.3097$) and the case of short-range scattering by uncharged impurities or quasi-elastic scattering by acoustic phonons ($\alpha = -\frac{1}{2}$, $P/P_{\text{Poisson}} = 0.3777$). The results in the drift approximation (24) are about 10% larger. Our values are somewhat smaller than those following from the numerical simulations of González *et al.*, who found $P/P_{\text{Poisson}} = \frac{1}{3}$ for $\alpha = 0$ [4] and $P/P_{\text{Poisson}} = 0.42 - 0.44$ for $\alpha = -\frac{1}{2}$ [12].

Our considerations require the exponent α to be in the range $-\frac{1}{2} < \alpha < 1$. For $\alpha < -\frac{1}{2}$ the mean free path $l \propto \varepsilon^{\alpha+1/2}$ diverges at small kinetic energies. The carriers at the current source therefore enter the conductor ballistically and accumulate only at a finite distance from the injection point. Fig. 1 indicates that the charge density at the current source must be zero if one insists that the electric field vanishes. Nagaev [8] has shown that full shot noise, $P = P_{\text{Poisson}}$, follows for $\alpha = -\frac{3}{2}$. Presumably, P/P_{Poisson} will decrease monotonically from 1 for $\alpha = -\frac{3}{2}$ to 0.38 for $\alpha = -\frac{1}{2}$, but we have no theory for this range of α's. For $\alpha > 1$ the resistance R becomes infinitely large, because the coefficient $\bar{\chi}(1)$ in the current-voltage characteristic (15) diverges. An intuitive understanding can be obtained by equating the potential gain $\phi \sim (Dt)^{3/(2\alpha+4)}$ (acquired by diffusing close to the current source for a time t) with the increase in kinetic energy ε: For $\alpha > 1$ this time $t \propto \varepsilon^{(1-\alpha)/3}$ is seen to diverge for small ε. We found that the shot-noise power vanishes as $\alpha \to 1$. Presumably, a non-zero answer for P would follow for $\alpha > 1$ if the non-zero thermal energy and finite charge density at the current source is accounted for. This remains an open problem.

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References

[1] M. J. M. de Jong and C. W. J. Beenakker, in: *Mesoscopic Electron Transport*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön, NATO ASI Series E345 (Kluwer, Dordrecht, 1997).

[2] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B **46**, 1889 (1992).

[3] K. E. Nagaev, Phys. Lett. A **169**, 103 (1992).

[4] T. González, C. González, J. Mateos, D. Pardo, L. Reggiani, O. M. Bulashenko, and J. M. Rubí, Phys. Rev. Lett. **80**, 2901 (1998); T. González, J. Mateos, D. Pardo, O. M. Bulashenko, and L. Reggiani, Phys. Rev. B in press (cond-mat/9811069).

[5] R. Landauer, Nature **392**, 658 (1998).

[6] C. W. J. Beenakker, Phys. Rev. Lett. **82**, 2761 (1999).

[7] H. Schomerus, E. G. Mishchenko, and C. W. J. Beenakker, Phys. Rev. B in press (cond-mat/9901346).

[8] K. E. Nagaev, preprint (cond-mat/9812357).

[9] Sh. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University, Cambridge, 1996).

[10] S. V Gantsevich, V. L. Gurevich, and R. Katilius, Rivista Nuovo Cimento **2** (5), 1 (1979).

[11] M. A. Lampert and P. Mark, *Current Injection in Solids* (Academic, New York, 1970).

[12] T. González, C. González, J. Mateos, D. Pardo, L. Reggiani, O. M. Bulashenko, and J. M. Rubí, private communication.