**Abstract**

Which are the underlying mechanisms of neuronal oscillations? Collective behaviors that arise in large networks of interacting neurons are known to play a crucial role in processing and coding of information in the brain. Here we analyze a network of Quadratic Integrate-and-Fire (QIF) neurons with delayed synaptic interaction. With a dimensionality reduction technique [1,2], we derive the exact firing rate equations for a population of identical neurons, and we study numerically and analytically the phase diagrams for both excitatory and inhibitory coupling. For inhibitory networks, we detect a novel region of oscillations, called quasiperiodic partial synchronization [3], and relate it with fast neuronal oscillations.

**Network description**

**Quadratic integrate-and-fire (QIF) neuron model**

\[ \tau \dot{V}_i = -V_i + I_i + J(D) \]

- **Membrane Potential**
- **Input current**
- **Time delay**

**Firing rate equations (FREs)**

**Derivation of mean field equations**

\[ \tau \dot{\nu} = 2r \nu + \frac{\Delta}{\tau} \]

- **Thermodynamic limit:** \( N \to \infty \)
- **Lorentzian distribution of voltages with width \( \Delta \) and mean \( \nu \)**

**Phase diagrams**

**Identical Neurons:** \( \Delta = 0 \)

The linear stability analysis of the splay state gives the boundaries:

- **Incoherence boundaries**
- **Synchronization boundaries**

**Effect of Heterogeneities:** \( \Delta \neq 0 \)

We derived the exact firing rate equations for a network of all-to-all coupled QIF neurons with delayed synaptic interactions. The interplay between inhibition and synaptic delay is confirmed to be an important mechanism of generation of complex oscillatory pattern. We reported the existence of a new oscillatory state for inhibitory coupling, that can be related with fast brain oscillations.