Analysing brain dynamics with a novel mutual information estimator

phase, power and their representational interactions

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Mutual Information (MI)

- Statistical test for dependence with a meaningful effect size (bits)
- In general sensitive to any type of dependence
- Variables can be discrete / continuous; uni- / multi-dimensional
- Additive effect size: allows for higher-order quantities (more than two variables)

Mutual Information (MI)

- Entropy (*H*) : a measure of uncertainty / spread / dispersion (c.f. variance)

$$I(X;Y) = H(X) - H(X|Y)$$

$$I(X;Y) = H(Y) - H(Y|X)$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

Estimating MI : Binning

- Discretise continuous values (e.g. quartiles of the distribution of values)
- Use discrete formulation of MI (summing of probabilities)
- Problem of *bias* and the *curse of dimensionality*

Estimating MI : Continuous

- Kernel density estimation
- Nearest neighbour
- Parametric assumptions

Estimating MI: Gaussian Copula

- Any multivariate distribution can be split into marginal distributions and a copula that links them.

$$F(x, y) = C(F_x(x), F_y(y))$$

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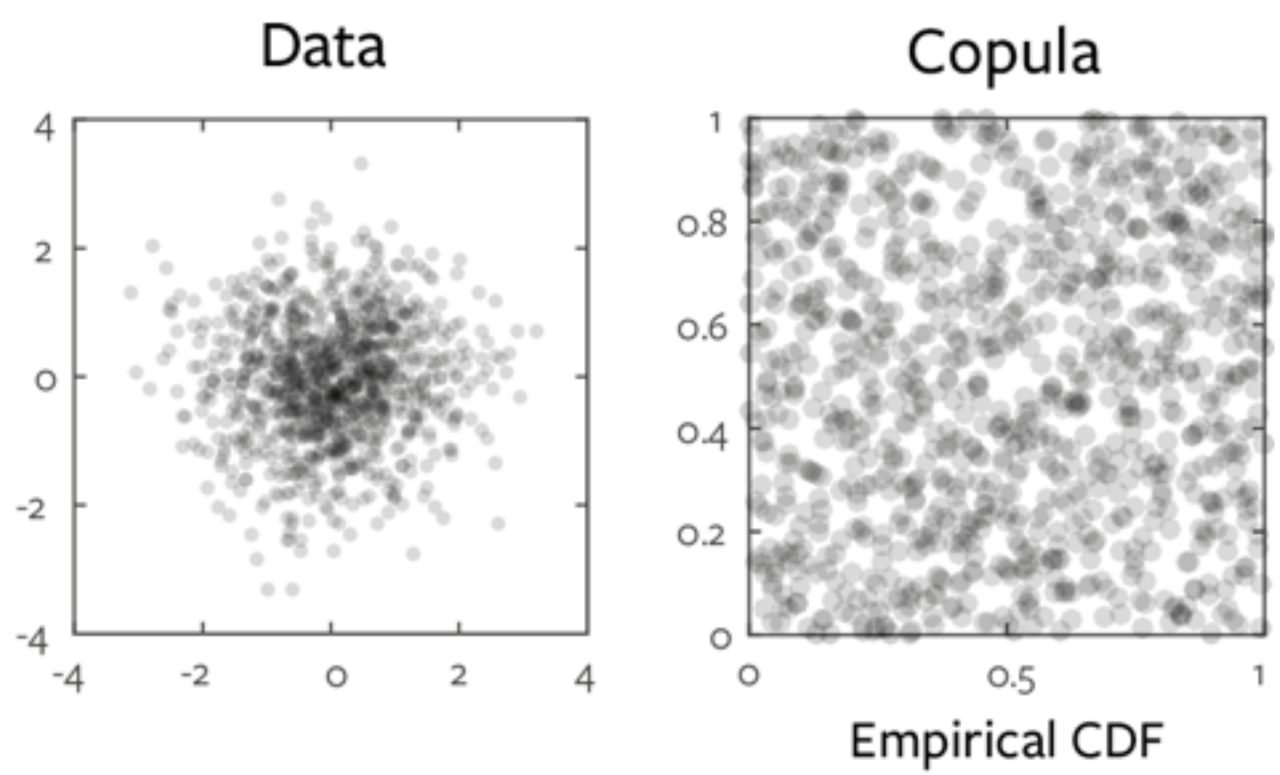

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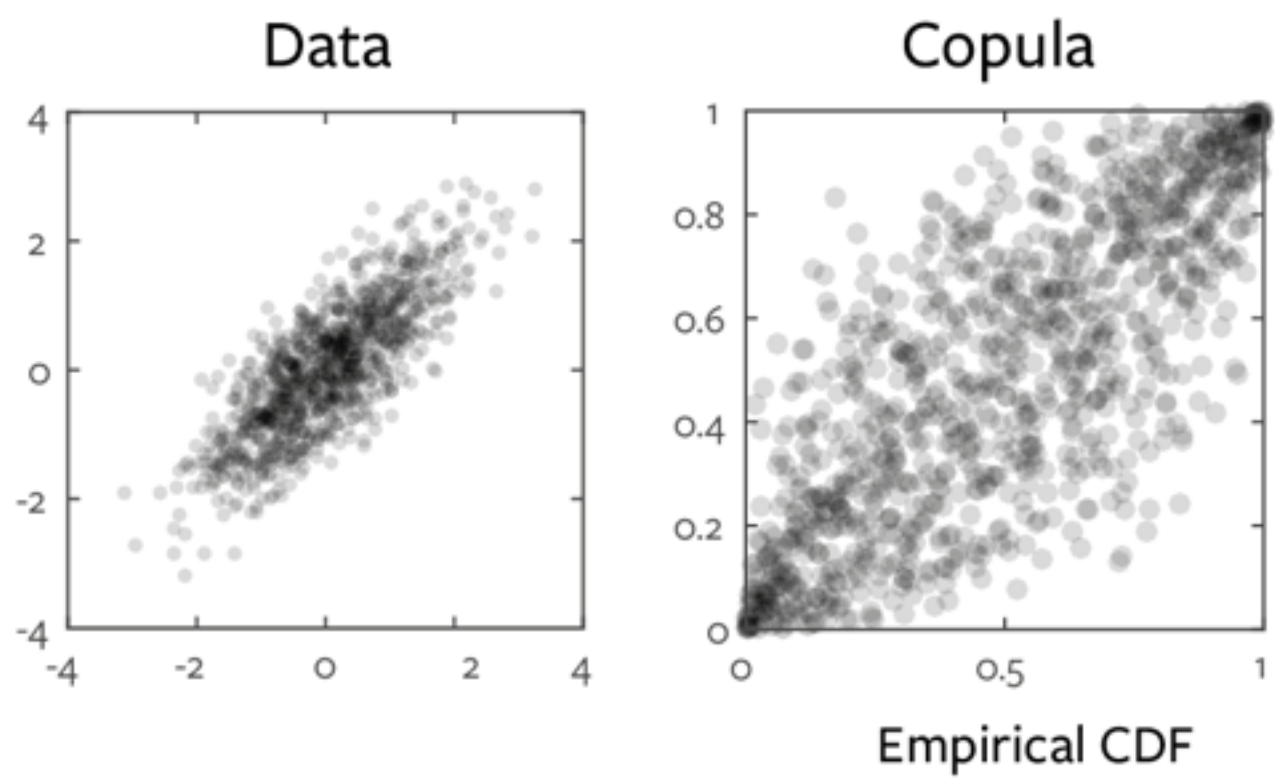

Estimating MI: Gaussian Copula

- Any multivariate distribution can be split into marginal distributions and a copula that links them.

$$F(x, y) = C(F_x(x), F_y(y))$$

- MI is a function only of the copula (Ma and Sun, 2011); does not depend on the marginals

- Semi-parametric approach: assume Gaussian copula (no assumption on marginals)

- Transform marginals to standard normal preserving empirical copula; apply Gaussian parametric estimation

Estimating MI: Gaussian Copula

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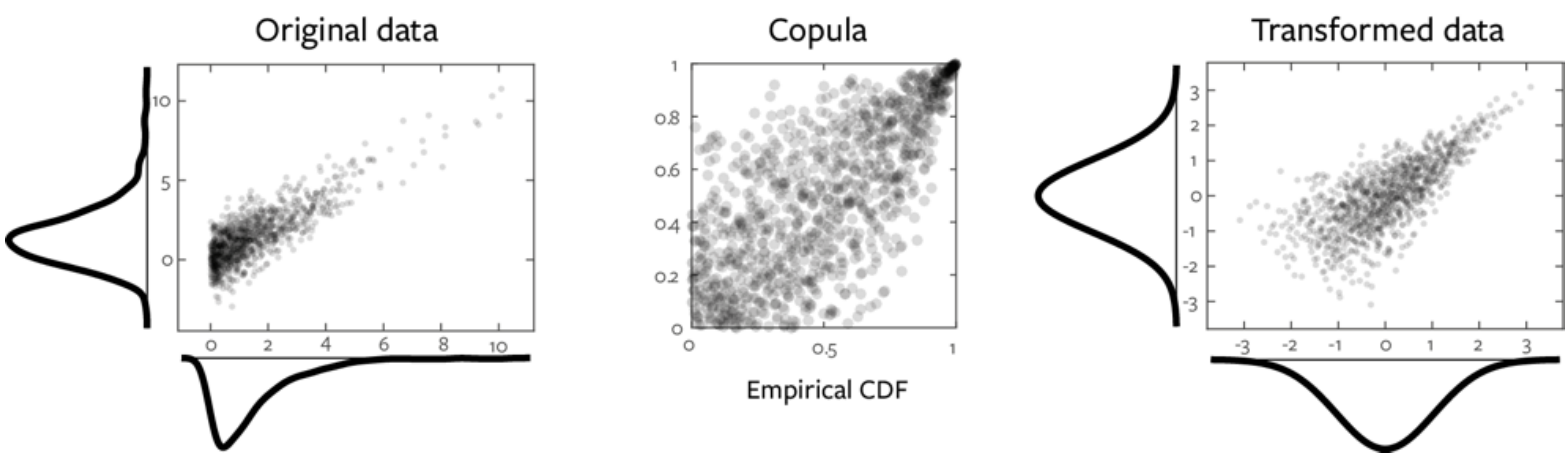


Estimating MI: Gaussian Copula

- Semi-parametric **lower bound** MI estimate
- Gauss-Copula Mutual Information (GCMI)
  https://github.com/robince/gcmi
- Think of it as a **multivariate** rank-correlation like statistic that can handle discrete and continuous variables and gives effect sizes on an **additive** common scale
- bioRxiv: http://dx.doi.org/10.1101/043745

Multivariate MI

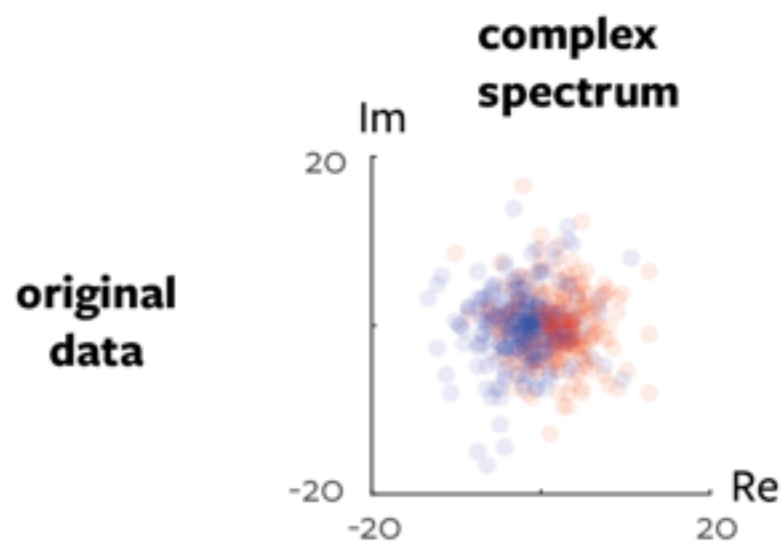
- For multidimensional variables, copula transform each dimension independently

- Can apply to low dimensional multivariate responses e.g. magnetic field vectors, EEG voltage + instantaneous temporal derivative, **complex spectra**

- Allows for higher-order information theoretic quantities : conditional mutual information, interaction information, directed information (transfer entropy), directed feature information

Spectral MI: phase and power

Simulation 1: Phase Modulation



Spectral MI: phase and power

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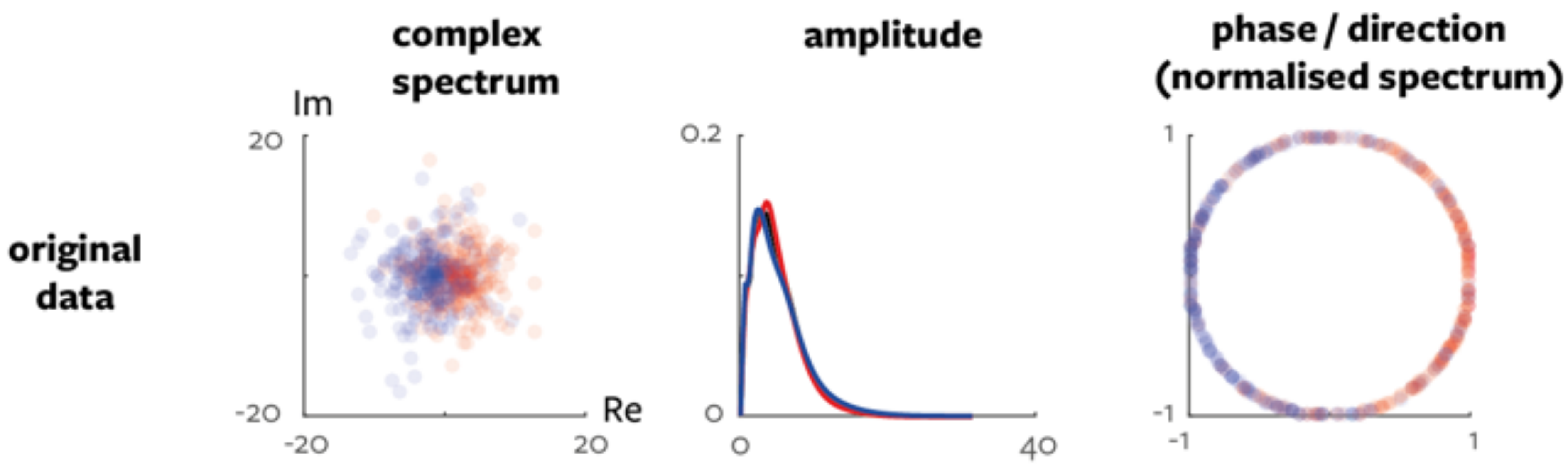

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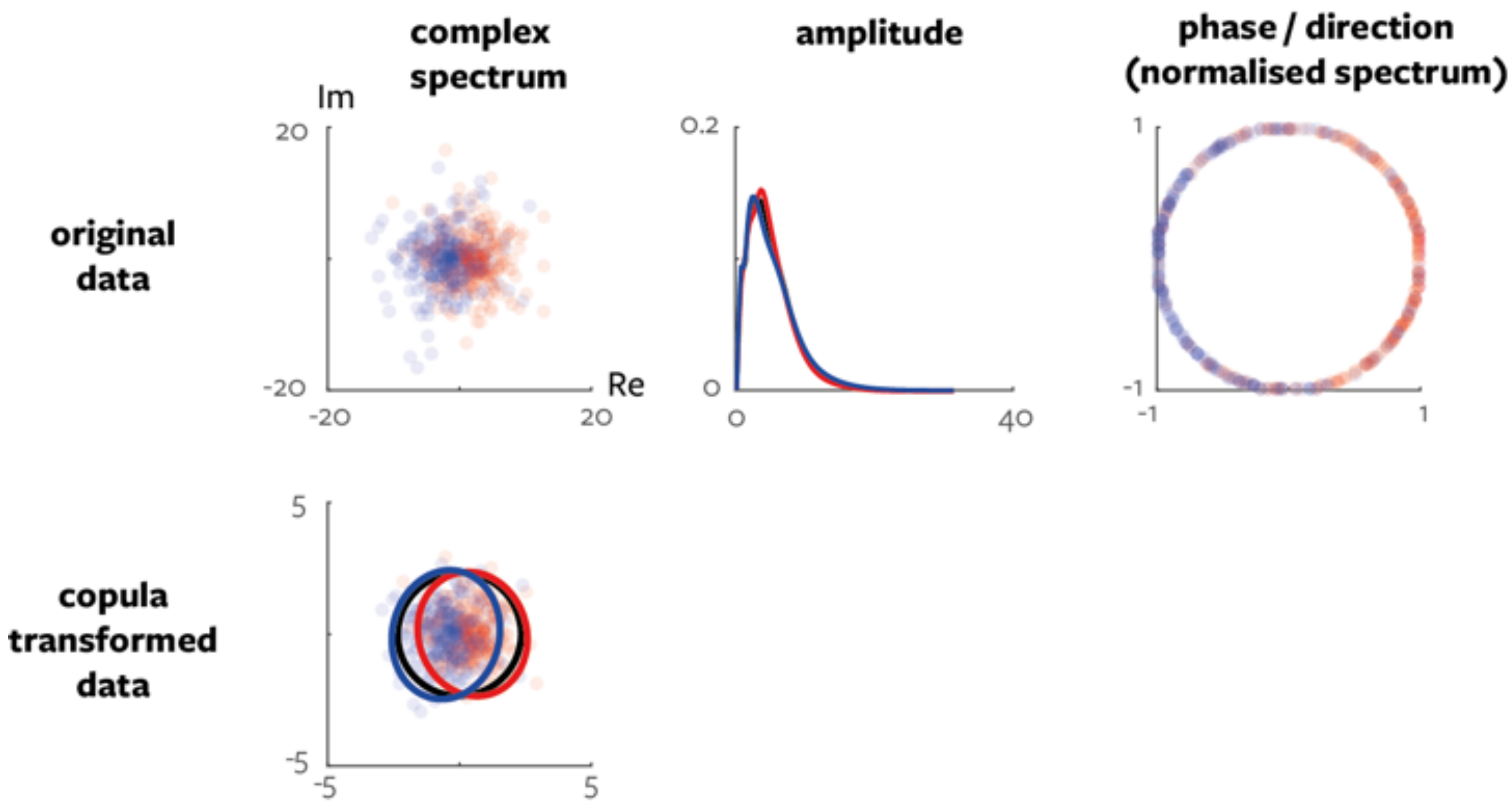

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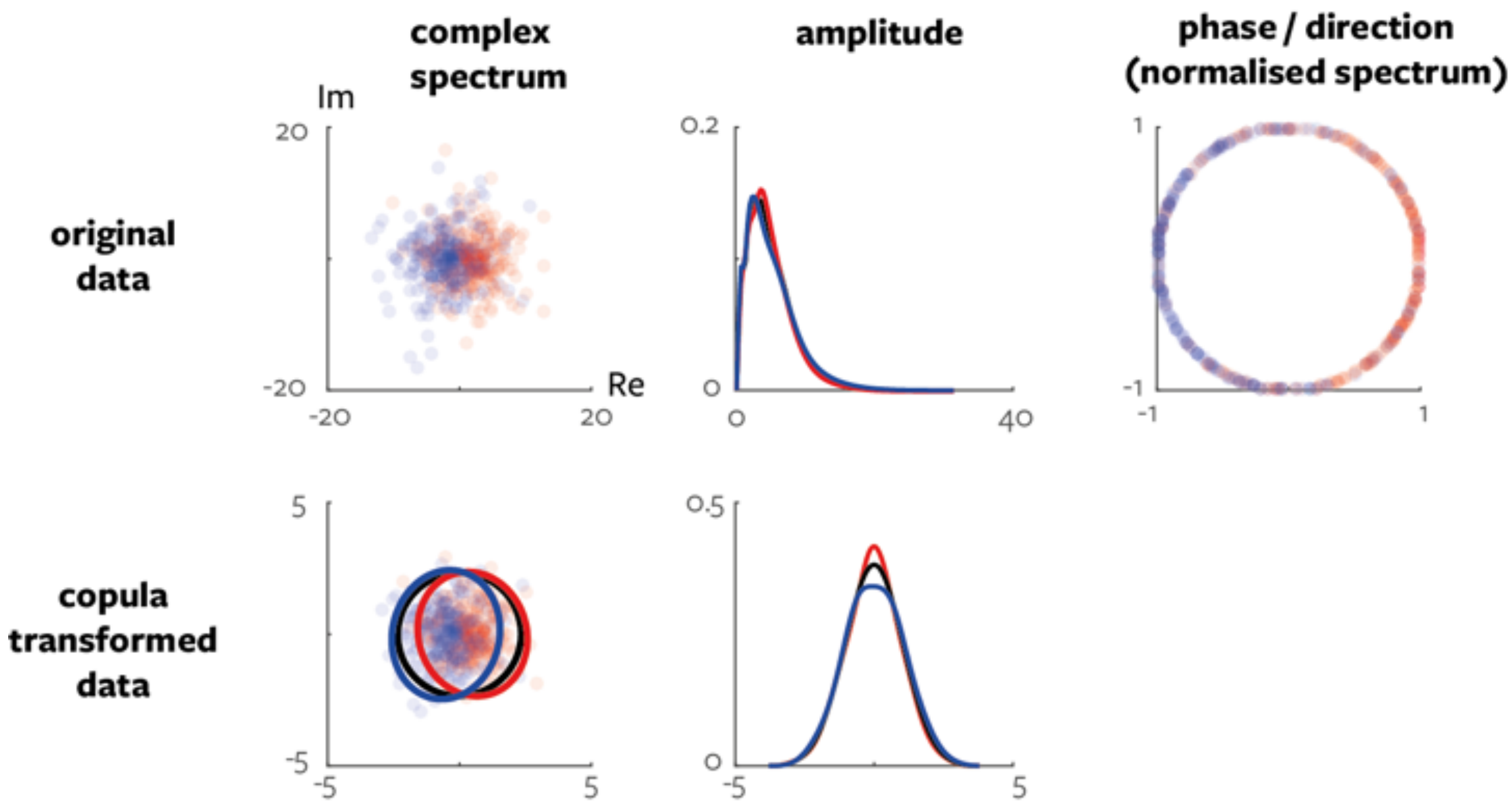

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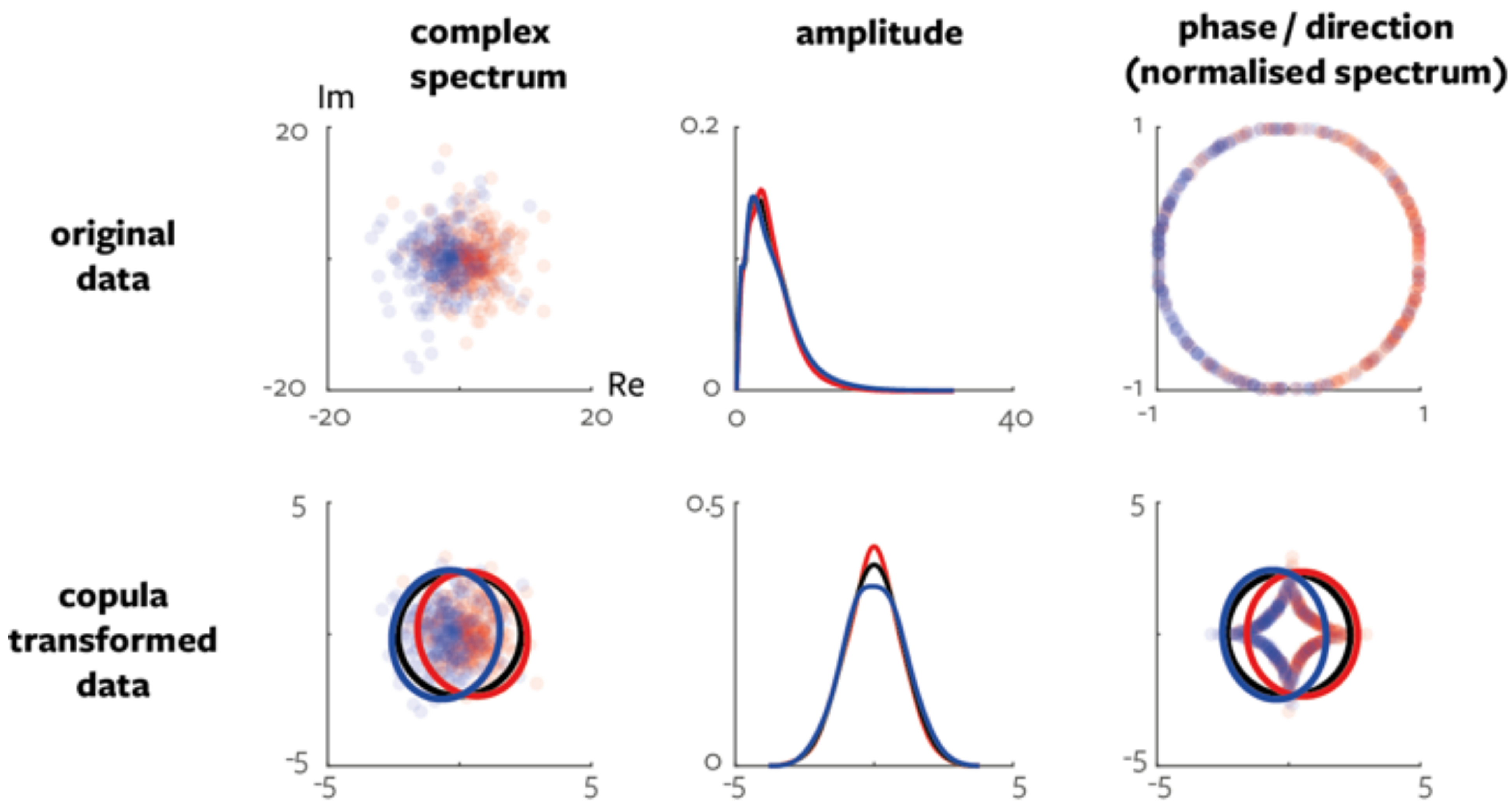

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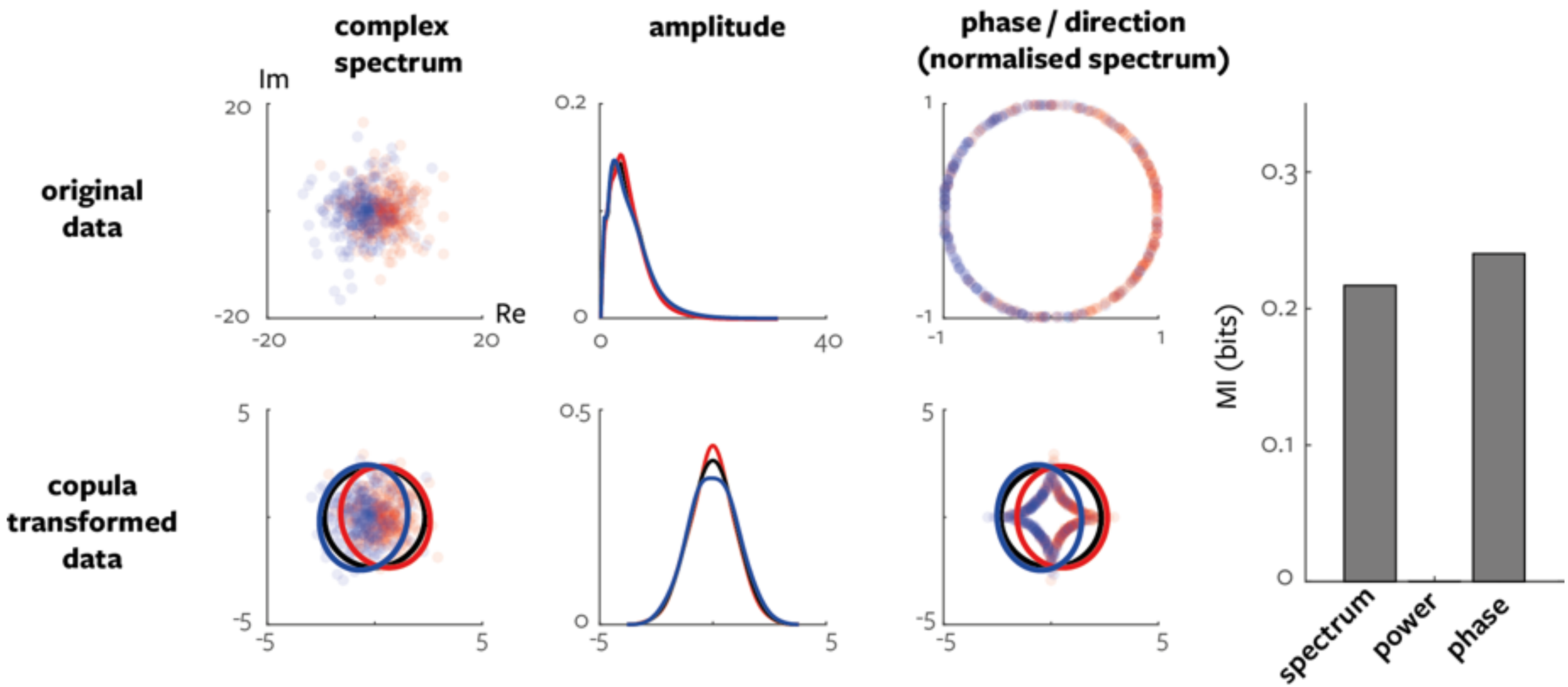
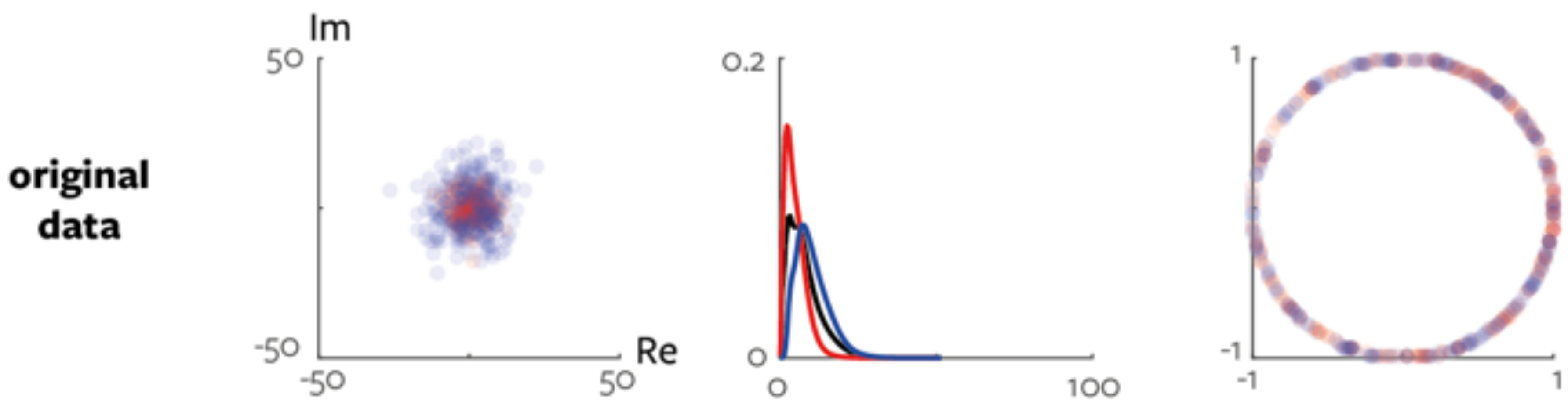


Spectral MI: phase and power

Simulation 2: Power Modulation



Spectral MI: phase and power

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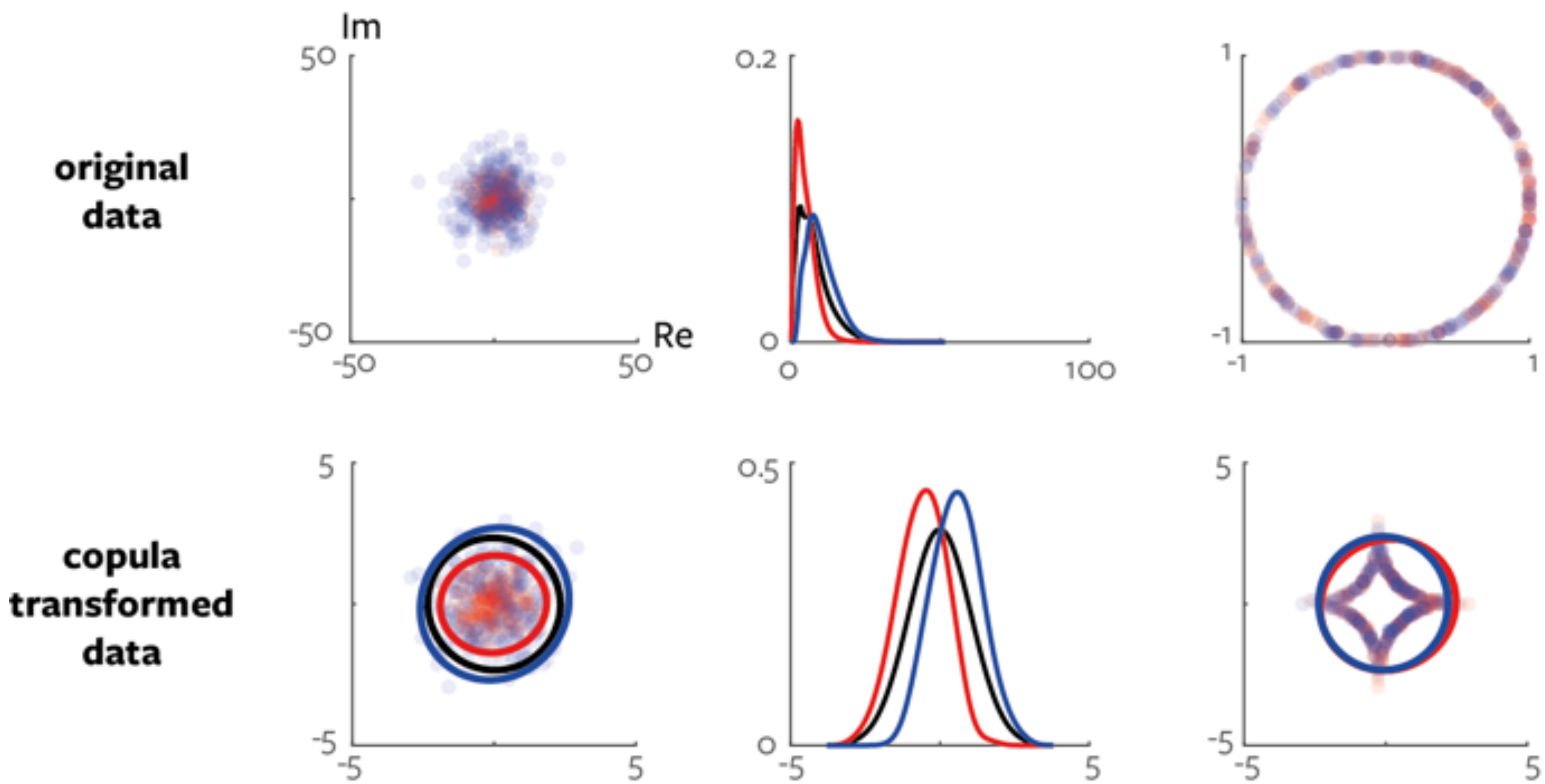

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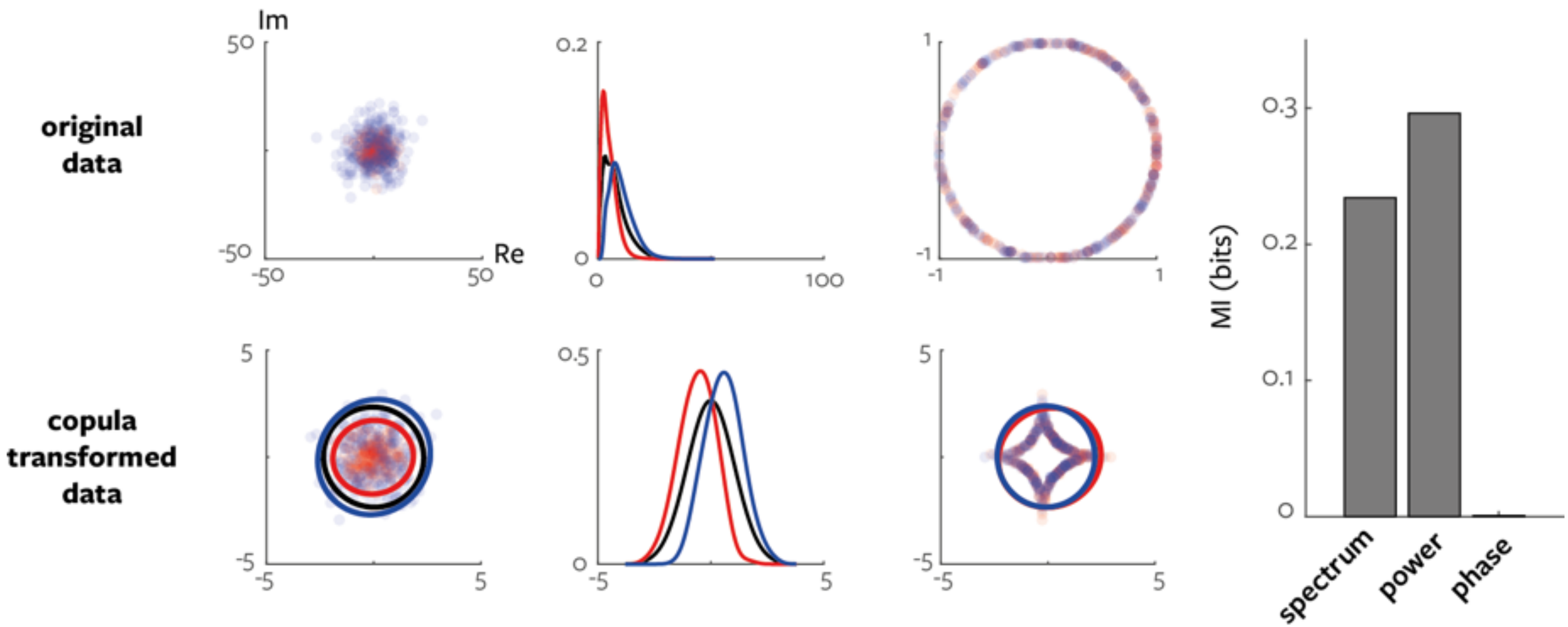

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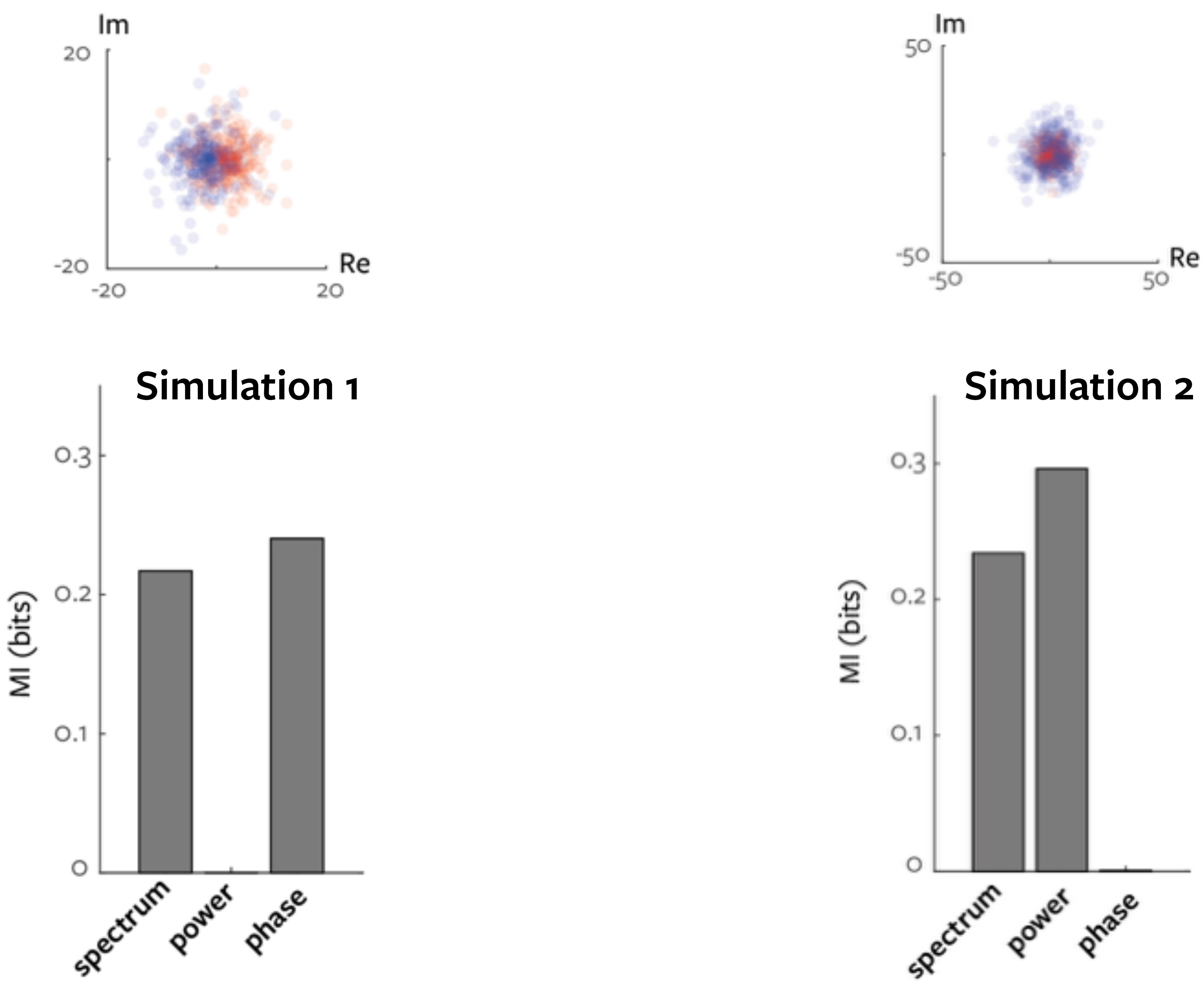


Spectral MI: phase and power

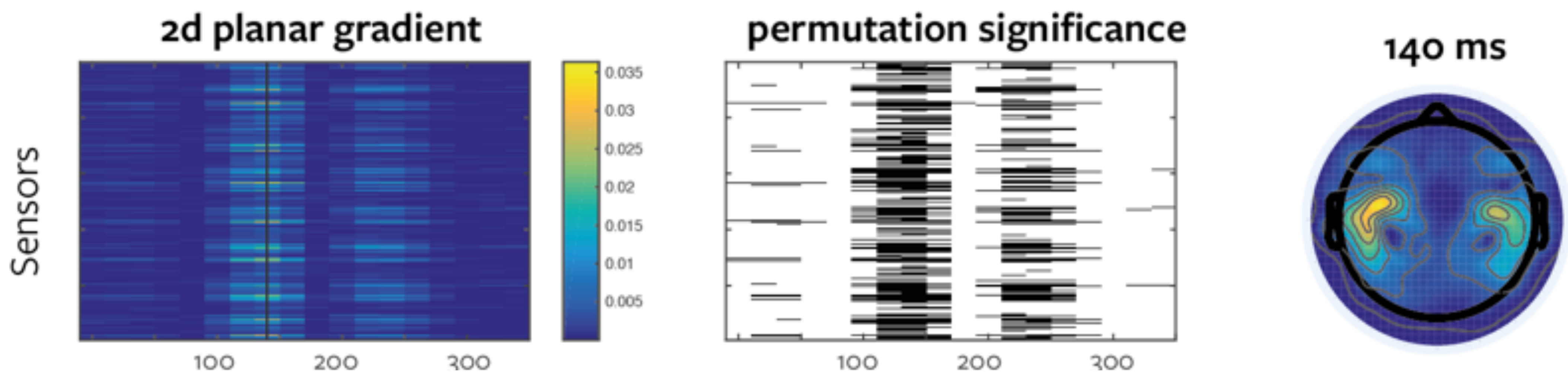
- Avoid issue of circular variables by remaining in 2D complex plane but normalising away effect of amplitude

- A test for modulation of phase + power by discrete or continuous experimental factors with a directly comparable effect size

- Can be applied to spectral data from any decomposition method (Hilbert, wavelets, emprical mode decomposition etc.)

- Interaction information : can directly relate modulations of phase and power within and across bands

Example: Planar magnetic field



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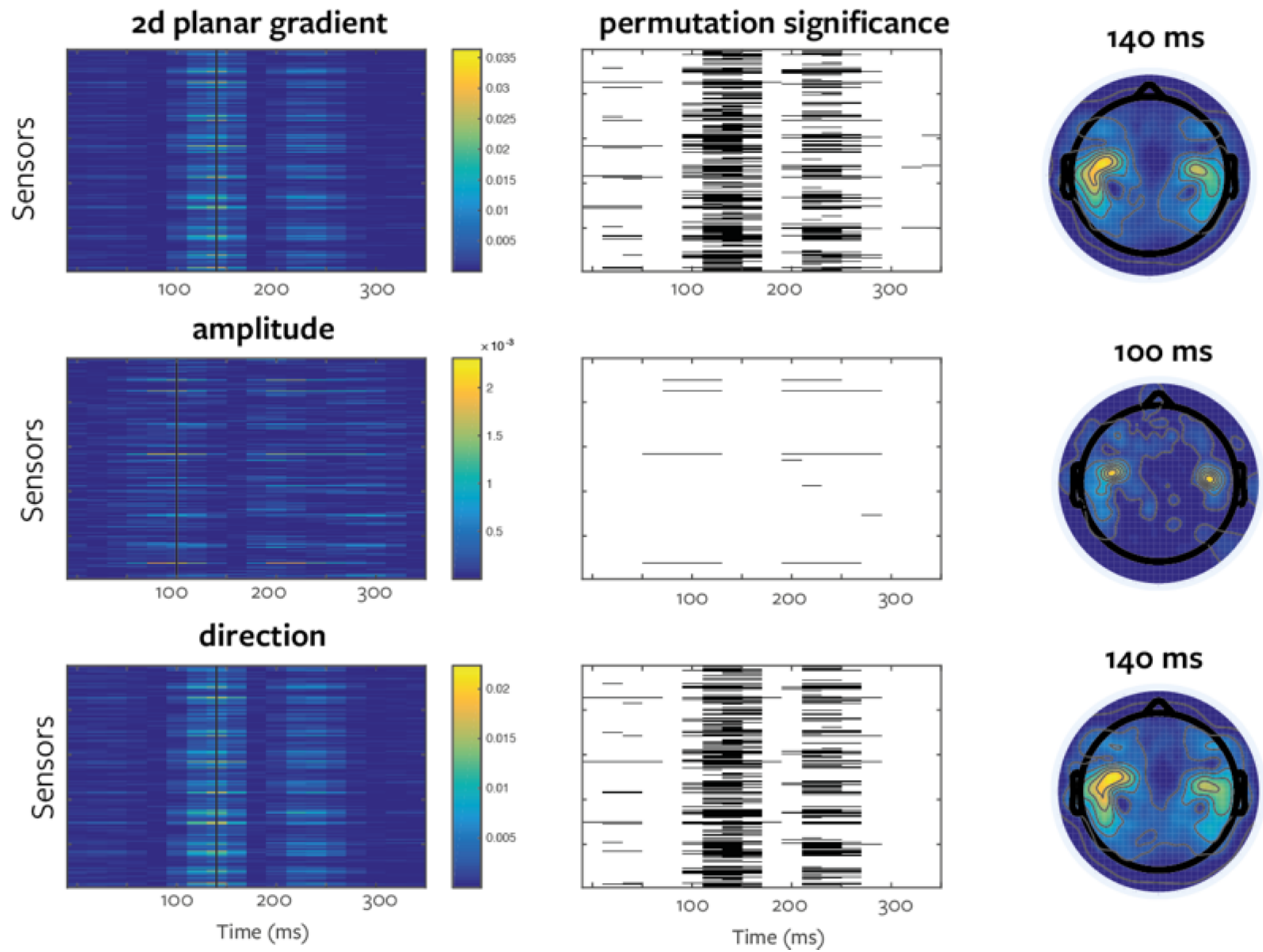
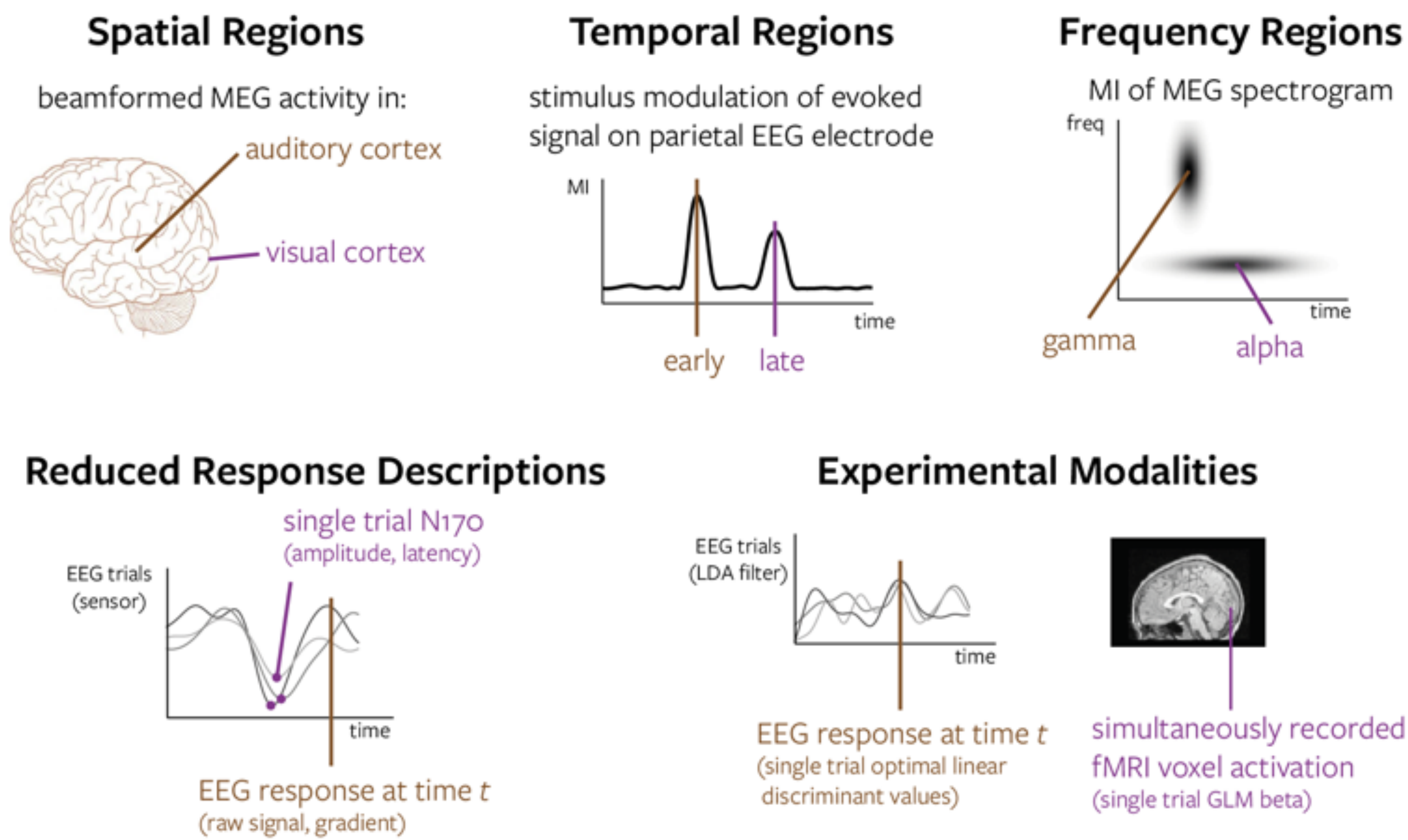
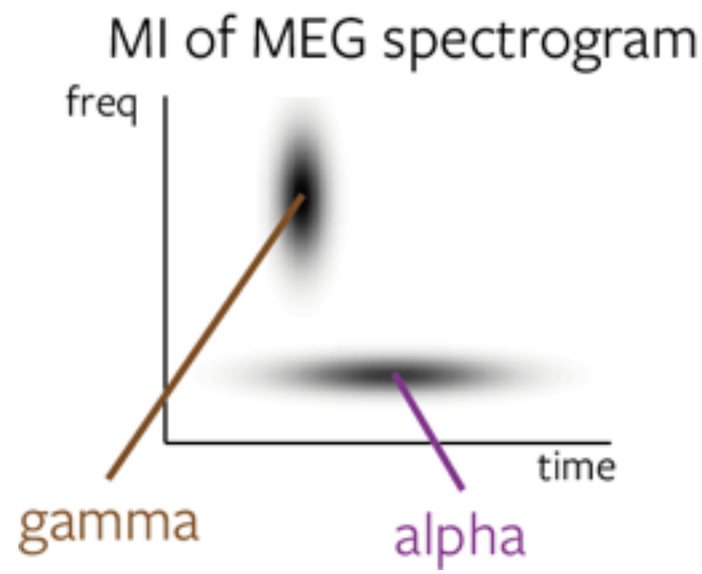


Representational Interactions



Interaction Information



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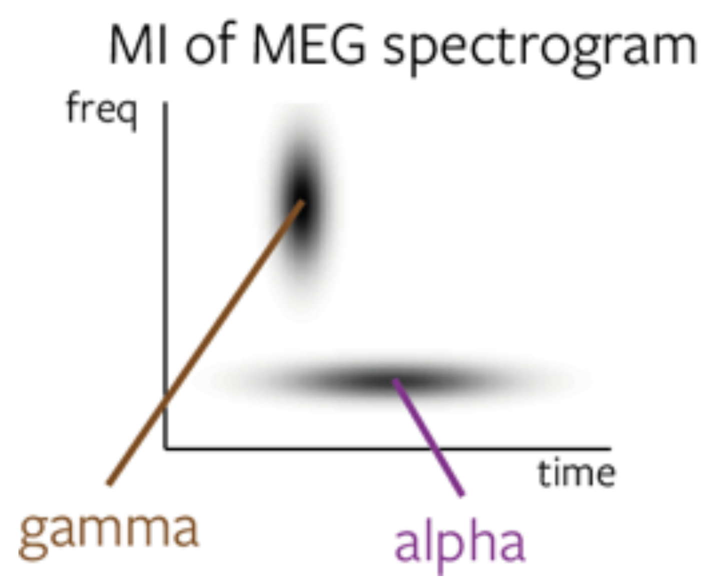

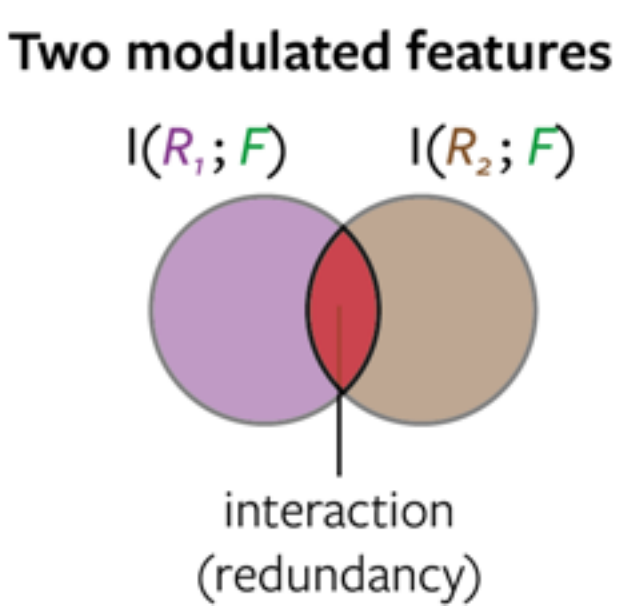

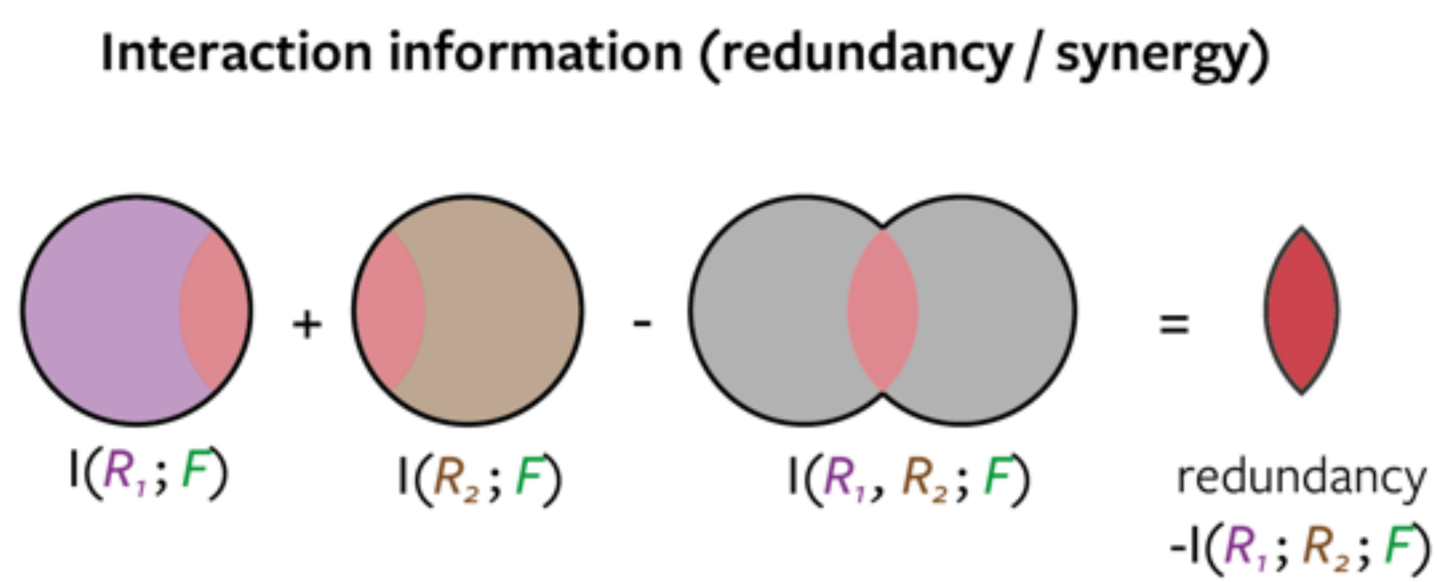

Interaction Information

- **Redudancy:** equivalent representation on a single-trial basis. Implies both bands reflect the same function / mechanism
- **Independence:** unrelated representations. Implies each band may reflect different processing mechanisms
- **Synergy:** the within trial relationship between the bands is itself modulated by the stimulus. Indicates stimulus modulated cross-frequency coupling.

Summary

- **GCMI** provides a multivariate rank-based statistical framework for data analysis
- Can be applied to spectral data to quantify and dissociate modulations of **phase and power** by discrete or continuous experimental factors
- Interaction Information can quantify **representational interactions** between phase/power, or frequency bands
- Can be used for causal / connectivity analysis : e.g. directed information between two different regions
- Can be used to quantify cross-frequency coupling : e.g. information between phase in one band and power in another band

Thanks

- github.com/robince/gcmi

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New Results

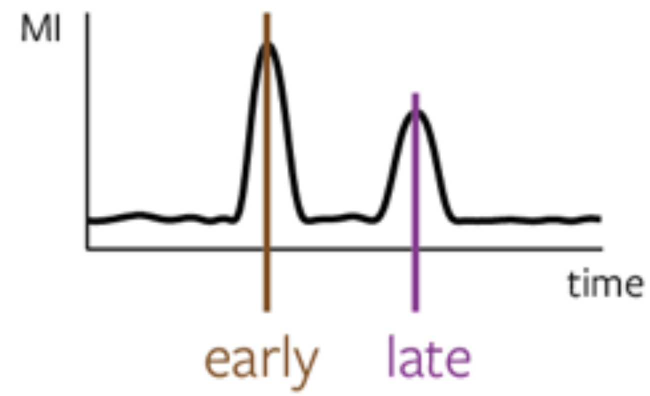
A statistical framework for neuroimaging data analysis based on mutual information estimated via a Gaussian copula

Robin A. A. Ince, Bruno L. Giordano, Christoph Kayser, Guillaume A. Rousselet, Joachim Gross, Philippe G. Schyns

doi: http://dx.doi.org/10.1101/043745

Example: temporal interaction

stimulus modulation of evoked
signal on parietal EEG electrode



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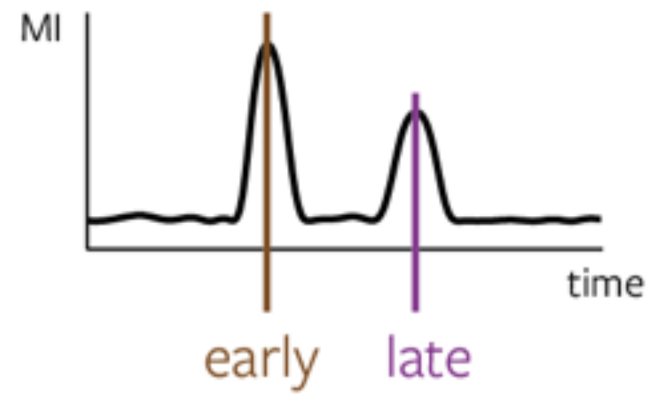
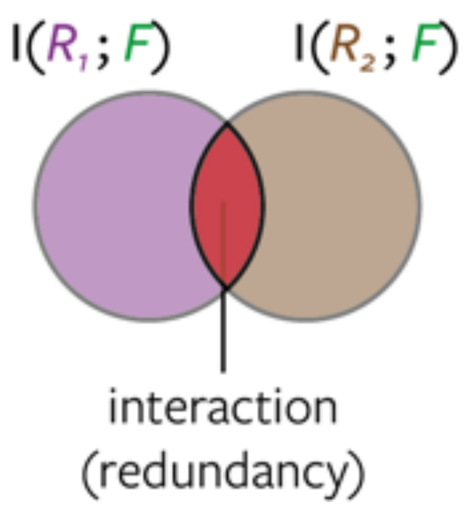
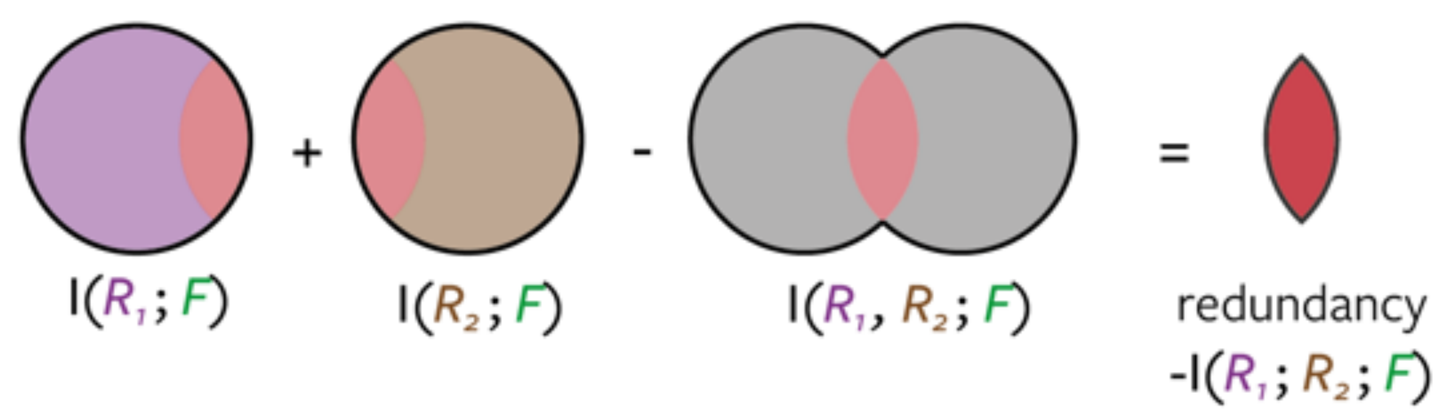


Two modulated features



Interaction information (redundancy / synergy)



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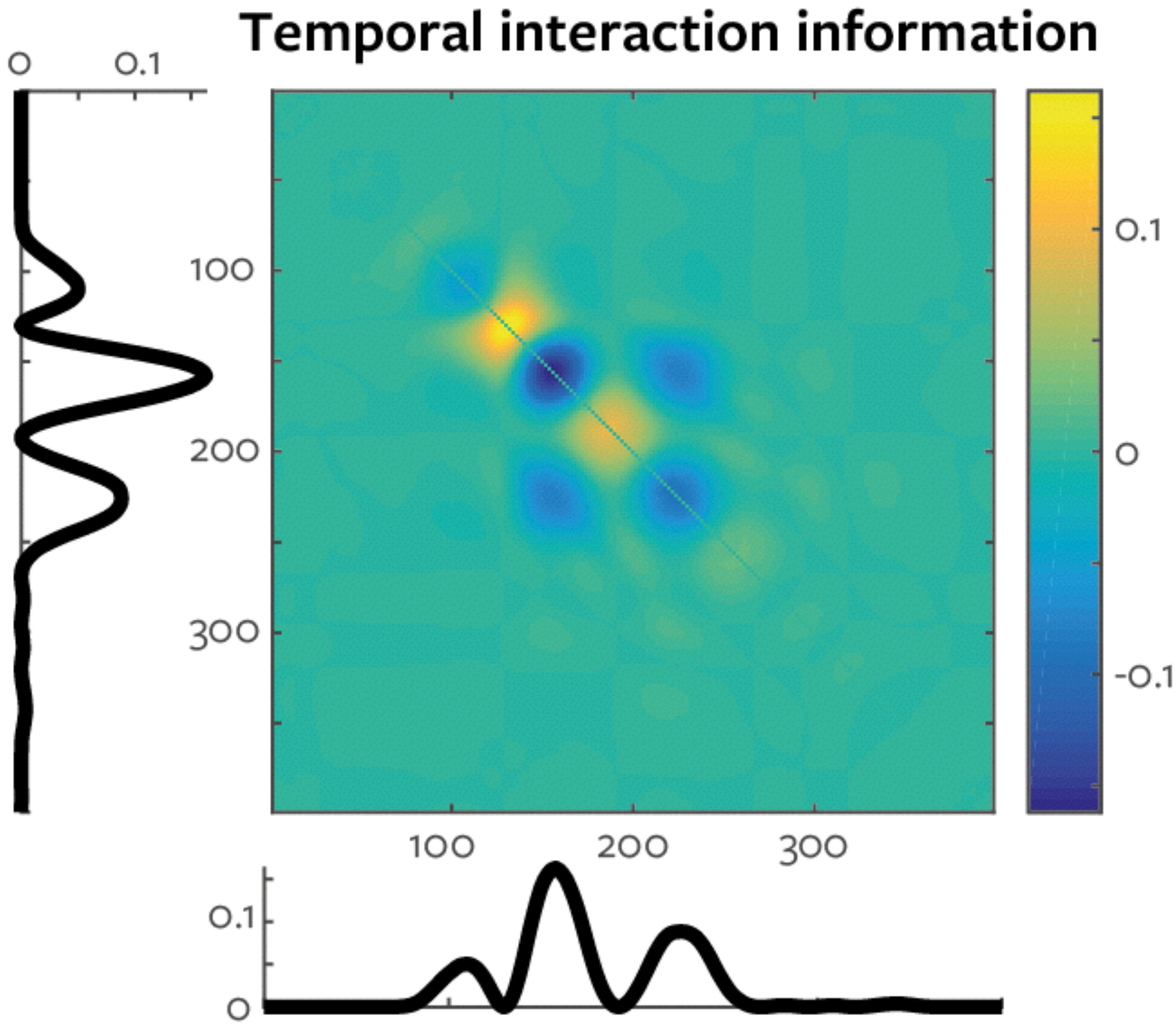

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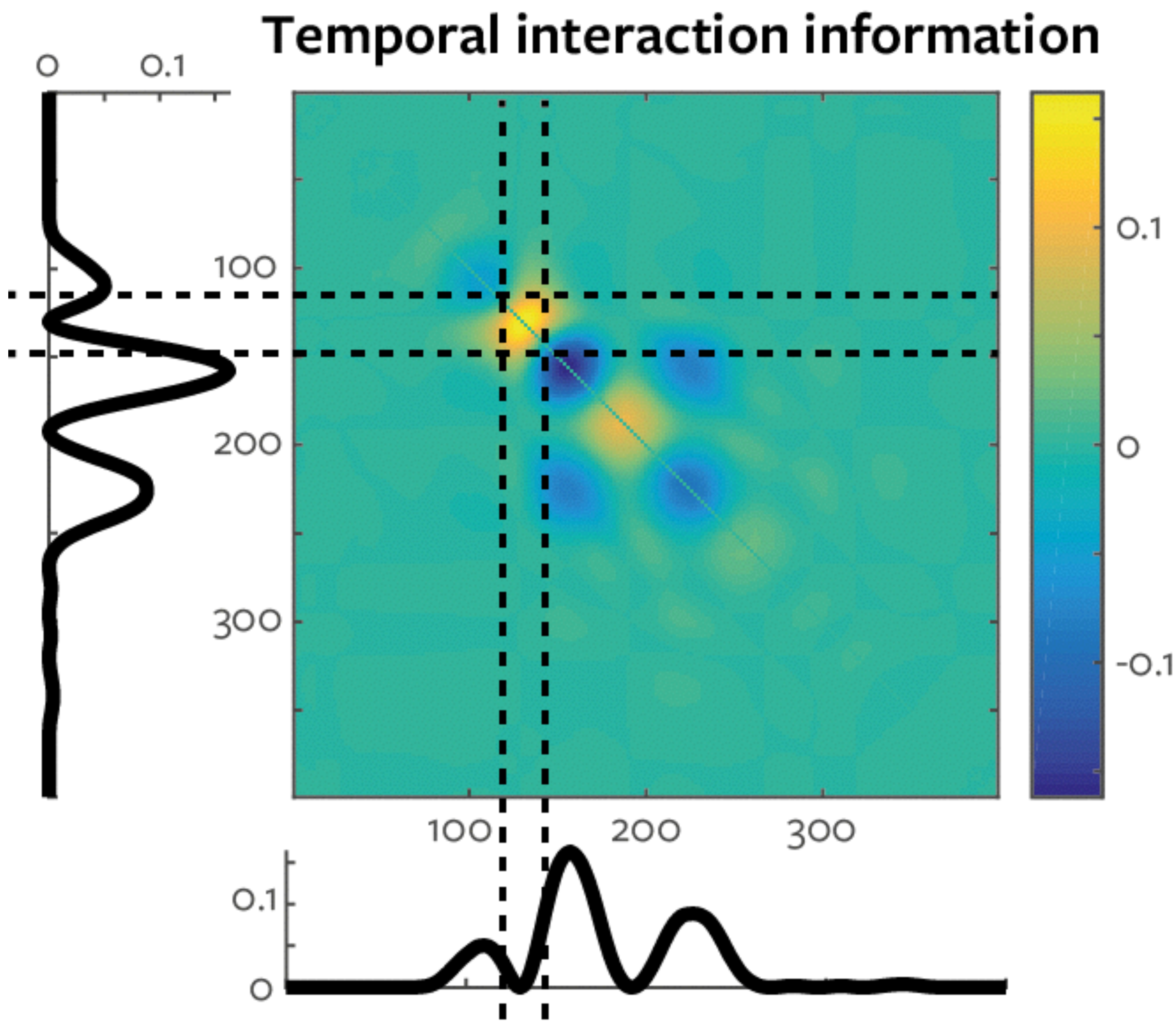
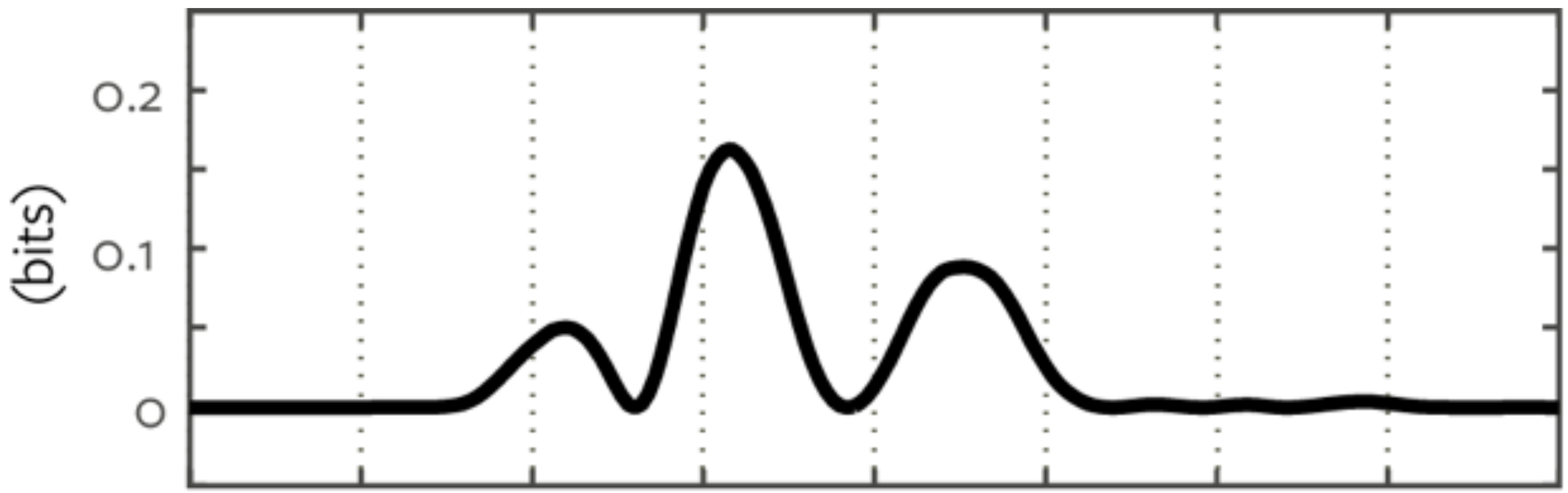
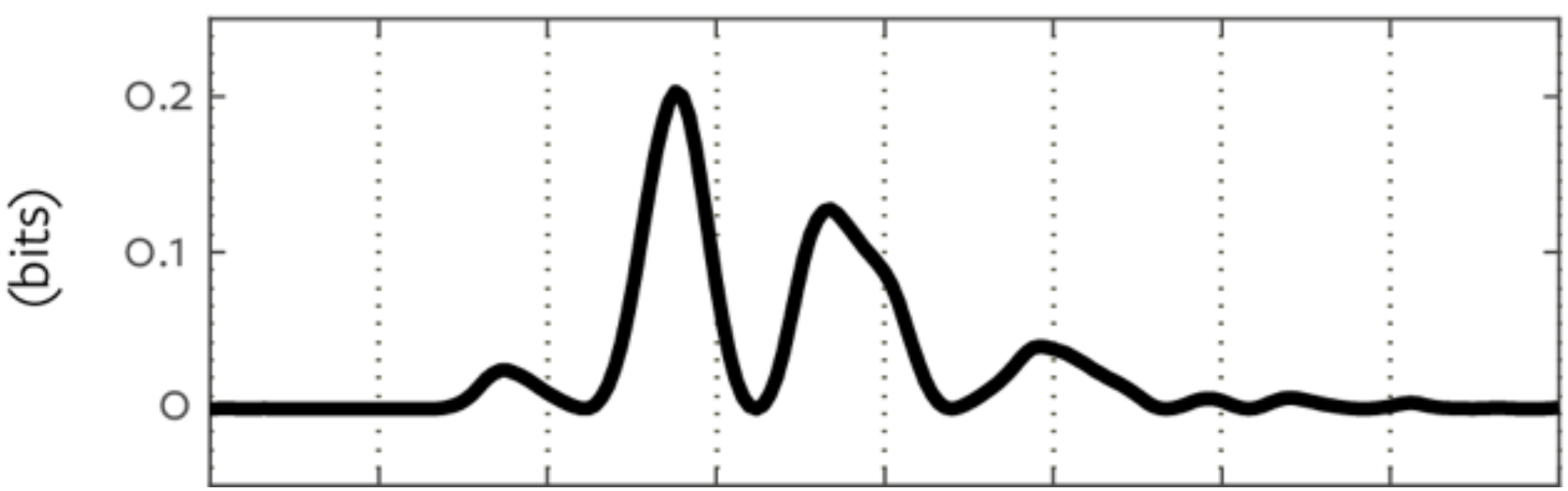


Example: gradient of voltage

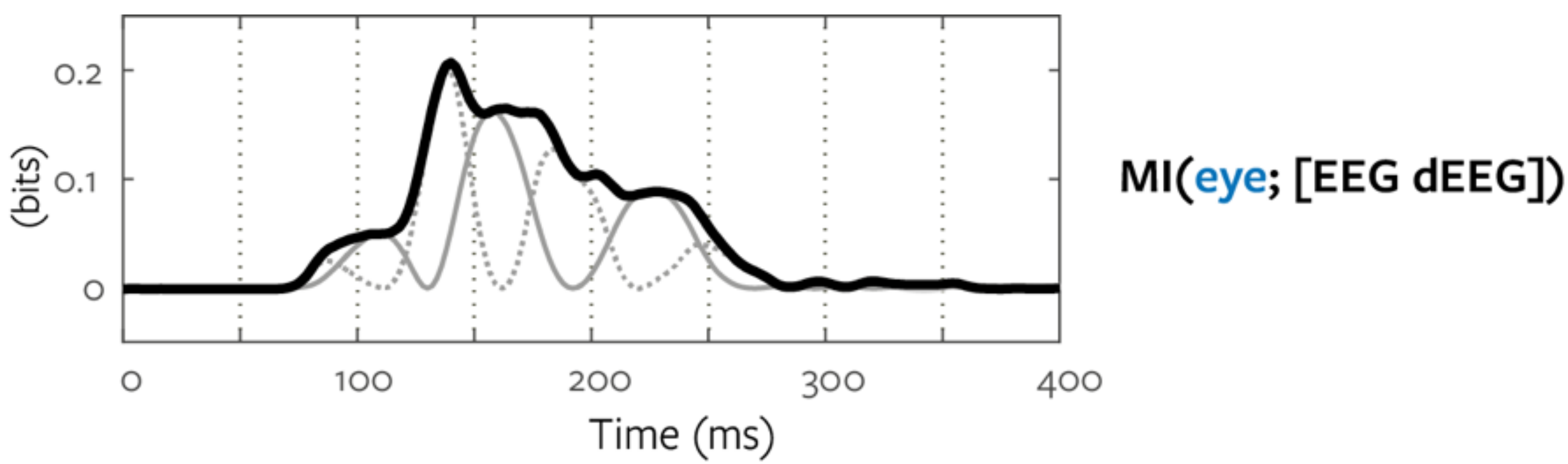


MI(eye; EEG)



MI(eye; dEEG)

Example: multivariate response (voltage, gradient)



Example: multivariate temporal redundancy