# Amplitude Change in $R$ and T Waves of Exercise Electrocardiogram 

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## Introduction

The exercise test is performed to evaluate the presence in the electrocardiogram (ECG) of myocardial ischaemia. In multistage Bruce protocol the patient on a bicycle ergometer is subject to a workload linearly increasing in time $(25 \mathrm{~W}$ every 2 minutes). The exercise is stopped when the heart rate reaches a maximum (acme). The standard 12-leads ECG is recorded using the electrocardiograph PC-ECG 1200 (Norav Medical Ltd.), with resolution of $2.441 \mu \mathrm{~V}$ and 500 Hz sampling frequency.

Exercise ECG


Figure 1: ECG at rest and acme (sampling units).

## Mathematical Model

The R and T waves amplitudes are measured from the ECG for each beat. These series are modeled by $X_{i}(t)=\mu(t)+Z_{i}(t)$ where $i=$ $1, \ldots, n$ is the index of the subject; $t=1, \ldots, m$ is the time index (beat number); $\mu(t)$ is the population mean; $Z_{i}(t)$ are independent realizations of the processes $Z(t)$, with zero mean, Var $Z(t)=\sigma(t)^{2}$ and covariance matrix $C$, representing error measurement and random individual deviations from the population mean.
The series are normalized dividing by their temporal mean to show relative variation during time. Each series $X_{i}$ is smoothed expanding it in an orthogonal basis of splines, giving $X_{i}^{(s)}$.

The estimated population mean is $\hat{\mu}(t)=$ $\sum_{i} X_{i}^{(s)}(t) / n$ and the estimated variance of $Z(t)$ is $\hat{\sigma}(t)^{2}=\sum_{i}\left(X_{i}^{(s)}(t)-\hat{\mu}(t)\right)^{2} /(n-1)$.

The simultaneous confidence band of level $1-\alpha$ of the population mean $\mu(t)$ is

$$
\hat{\mu}(t) \pm \hat{\sigma}(t) M_{1-\alpha} n^{-1 / 2}
$$

where $M_{1-\alpha}$ is the quantile of the r.v.

$$
M=\max _{t=1, \ldots, m}|Z(t)| / \hat{\sigma}(t)
$$

The resulting bands are in fig. 3 (left).

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## SiZer - Significant Zero Crossing of derivative

This method estimates the population mean $\mu(t)$ of raw data series and its derivative using a model 'trend plus noise' $Y(t)=\mu(t)+W(t)$ where $Y(t)=\frac{1}{n} \sum_{i=1}^{n} X_{i}(t)$ and the noise is $W(t)=\frac{1}{n} \sum_{i=1}^{n} Z_{i}(t)$.
A sequence of second order polynomials $P_{j}(t)=\beta_{0}^{(j)}+\beta_{1}^{(j)}\left(t-t_{j}\right)+\beta_{2}^{(j)}\left(t-t_{j}\right)^{2} ; j=1, \ldots, k$ is fitted locally to $Y(t)$ by minimization of the cost function $\sum_{t}\left(Y(t)-P_{j}(t)\right)^{2} w_{j}(t) ; w_{j}(t)=$ $K\left(\left(t-t_{j}\right) / h\right) / \sum_{t} K\left(\left(t-t_{j}\right) / h\right)$ is a Gaussian kernel and the bandwidth $h$ acts as a smoothing parameter.
The theory of multivariate linear models provide both an estimate and a confidence interval for the polynomial coefficients $\beta^{(j)}$. The coefficients $\beta_{0}^{(j)}, \beta_{1}^{(j)}, j=1, \ldots, k$ provide respectively an estimator of $\mu(t)$ and of its derivative. Their confidence intervals allow to construct the confidence bands as in fig. 3 (right).

## Results



Figure 2: Functional data of: $R R$ interval (top), R wave amplitude (middle), T wave (bottom) and their population means (thick black line) in a window centered at the acme (vertical line); x-axis: beat number; y -axis: RR ms ; R and T amplitude normalized units.


Figure 3: Left: Simultaneous confidence band of R and T population means, compared to the constant line of height 1 in a window centered at the acme (vertical line). Right: R and T population means (top) and zero crossing of the derivative (bottom) with confidence bands; $y$-axis: normalized untis.

